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# Modeling how fast teacher trainees master statistical concepts in verification of the learning curve theories

Richard Ayamah [1,*], Adolf Odei [1], Patrick Kwame Babah [2] and Emmanuel Teku [3]

[1] *Mathematics/ICT Department, St. Joseph's College of Education, Bechem-Ahafo Region, Ghana.*
[2] *Mathematics/ICT Department, Ada College of Education, Ada-Greater Accra Region, Ghana.*
[3] *Mathematics/ICT Department, Berekum College of Education, Berekum-Bono Region, Ghana.*

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## Abstract

This study was undertaken to model and evaluate how fast Ghanaian teacher trainees master statistical concepts, in verification of the learning curve theory. The target population was all the 2020/2021 academic year level 300 teacher trainees in Ghana. However, St. Joseph's College of Education (JOSCO), Bechem, was selected out of proximity from the 46 colleges of education in Ghana. At JOSCO, the Science/Mathematics, Agriculture/ICT and the Social Studies classes were randomly selected from the five level 300 classes and all the students in the three classes served as the experimental units. In effect, a ten-week period experimental tuition and evaluation on Educational Statistics was conducted and the data on the percentage of statistical concepts mastered by the teacher trainees were collected for the study. Tabular and graphical analyses on the data were carried out for the verification of the learning curve theory. Ordinary differential equations were used to derive the model for the prediction of the percentage of statistical concepts mastered by the teacher trainees. Graphical and numerical residual analyses were also conducted for the model adequacy checking. In all, the learning curve was successfully verified and the derived model was also found to be adequate for the prediction of the percentage of statistical concepts mastered by the teacher trainees at any point in time. This happens to be the first time research has been successfully conducted to verify the learning curve theory in statistical concept build-up in the colleges of education in Ghana.

**Keywords:** Statistical Concepts; Teacher Trainees; Verification; Learning Curve Theory; Time; Concave Down; Concave Up; How Fast; Increasing Rate; Decreasing Rate; Derived Model; Residual

## 1. Introduction

### 1.1. Overview of the Study

The research was conducted with the purpose of deriving a model for the evaluation and determination of how fast teacher trainees in Ghana learn statistics, and also to verify the learning curve theory.

The research was undertaken alongside the normal teaching schedules at St. Joseph's College of Education (JOSCO), Bechem, in the Ahafo Region in Ghana. The statistics course involved was Educational Statistics (with course code EBS348), mounted by the Institute of Educational Development and Outreach for Colleges of Education Studies under University of Cape Coast (UCC) in the second semester of the 2020/2021 academic year for all level 300 teacher trainees in all the forty-six public colleges of education in Ghana. The course outline entailed seven units of subject matter sequenced correspondingly as Nature of Statistics, Data Management and Representation, Measures of Scatter (Dispersion/Variation), Measures of Position, Measures of Relationships, Normal Distribution, Introduction to Hypothesis Testing and Simple Regression.

*Corresponding author: Richard Ayamah

### 1.2. Statistics and It's Origin

Statistics is the science of collecting, analyzing and presenting data in a more usable and understandable form. Statistics can be classified into two main branches, namely descriptive statistics and inferential statistics. Descriptive statistics is limited to collecting and describing important features of data. This covers such topics as Nature of Statistics, Data Management and Representation, Measures of Scatter (Dispersion/Variation), Measures of Position, etc. Inferential statistics is the analytical process taken to draw conclusions about a population of interest based on sample data from the population. This also covers such topics as Measures of Relationships, Normal Distribution, Introduction to Hypothesis Testing and Simple Regression, etc. [1]

Historians trace the origin of statistics to two roots which are rather dissimilar – games of chance and political science. In the seventeenth century, a French gambler, Chevalier de Mere, asked the famous mathematician and scientist, Blaise Pascal (1623-1662), to assess the odds involved in the game of chance played with dice. The attempt at solving de Mere's problem greatly contributed to the development of the theory of probability, which now forms the mathematical basis of statistics. In about the same period, interest in describing and analyzing political units brought a new area of demography which, for apparent lack of an existing name, was called 'Political Arithmetic'. The coining of the word 'statistik' is attributed to H. Conring (1606-1681), a professor at Hellmstadt University, Brunswick. However, the word began to be used in its modern sense only in the eighteenth century. Among the early users of this word is J.P Sussmilch (1707-1767), a German demographer. The pioneering works of James Bernoulli (1654-1705), K.F. Gauss (1777-1855), Karl Pearson (1857-1936), etc., greatly contributed to the development of the subject. [1]

### 1.3. Brief History of the Learning Curve Theory

The learning curve theory generally states that "a learner's efficiency in a task improves over time the more the learner performs the task". [2]

An early empirical demonstration of the learning curves was produced in 1885 by the German psychologist Dr. Herman Ebbinghaus. Ebbinghaus was investigating the difficulty of memorizing verbal stimuli. He found that performance increased in proportion to experience (practice and testing) on memorizing the word set. [3]

Later, Bills (1994) described the learning curve as a graphical device for picturing the rate of improvement in terms of a given criterion of efficiency, as a result of practice. He further described two sides of the same process, as learning curve of increasing and as eliminative/declininglearning curve, and had presented the corresponding two learning curve graphs as follows:

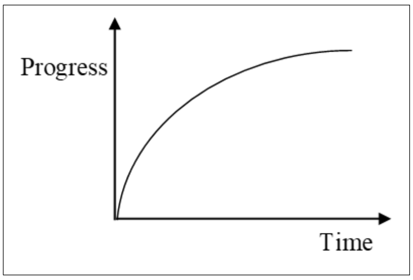


**Figure 1** Learning curve of increasing progress

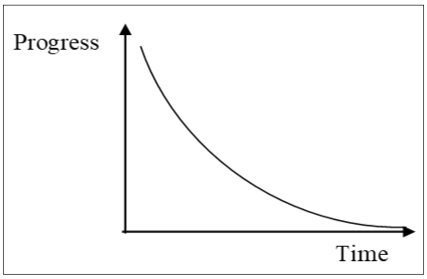


**Figure 2** Learning curve of eliminative/declining progress. [4]

The learning curve theory was later more generalized to: the more times a task has been performed, the less time is required on subsequent iteration. This was first quantified in the industrial setting in 1936 by Theodore Paul Wright. He found that every time the total aircraft production doubled, the required labour time for a new aircraft fell by 20%. He then formulated the experience curve theory which states that "the effort to complete a task should take less time and effort the more the task is done over time". [5]

### 1.4. What is Learning Curve

A learning curve is a correlation between a learner's performance on a task and the number of attempts or time required to complete the task; this can be represented as a direct proportion on a graph. [2]

According to 360 learning team, learning curve is the variance in the relationship between practice and proficiency over time. [6]

In reference [2], it is stated that "although the learning curve theory states: more attempts/time = a decrease in time, it does not always work out that way". Many factors can impact the end-results, resulting in a variety of different learning curve shapes, of which the four common types are as follows:

- The Diminishing-Returns Learning Curve: This is used to illustrate activities whereby the rate of progression increases rapidly at the beginning and then decreases over time. This situation describes a situation where the task may be easy to learn and progression of learning is initially fast and rapid. However, progression levels off as the learner obtain some high level of proficiency. This is just like Bill's learning curve of increasing progress, as in figure 1.

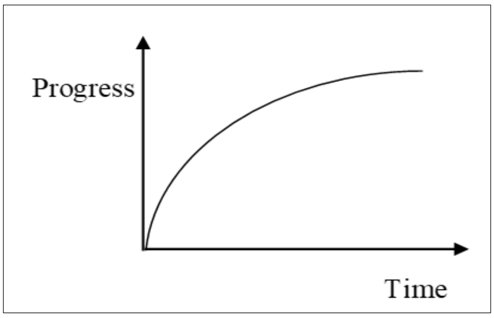


**Figure 3** Learning curve of diminishing-returns

- Increasing-Returns Learning Curve: This is used to illustrate activities whereby the rate of progression is slow at the beginning and then rises over time until full proficiency is obtained. This model describes a situation whereby a complex task is being learned and the rate of learning is initially slow but become faster over time.

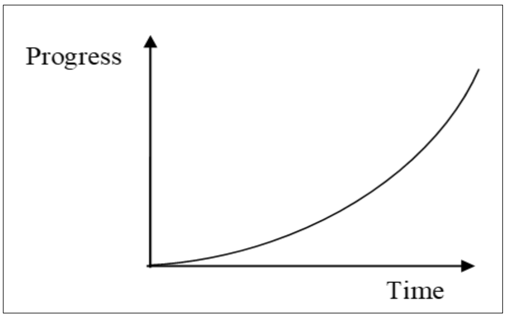


**Figure 4** Learning curve of increasing-returns

- The S-Curve: This is used to illustrate activities whereby the learner is new to the task and takes more time to master the skills but as time goes on the learner becomes proficient and takes less time in mastering the skills. However, the rate of progression levels off over time.

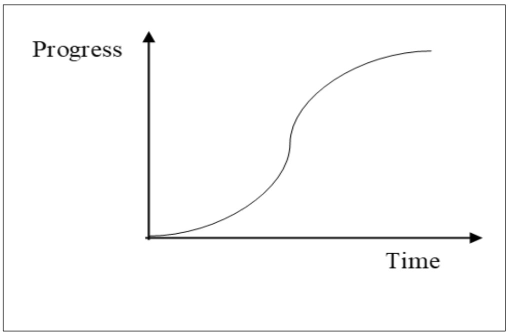


**Figure 5** Learning curve of increasing-decreasing-returns ($S$-curve)

- Complex Learning Curve: This is used to illustrate more complex learning journeys over a longer timeframe. Here, in the early stages, the learner's progression rate is a bit fast but becomes slower, then faster before leveling off at the latter stages.

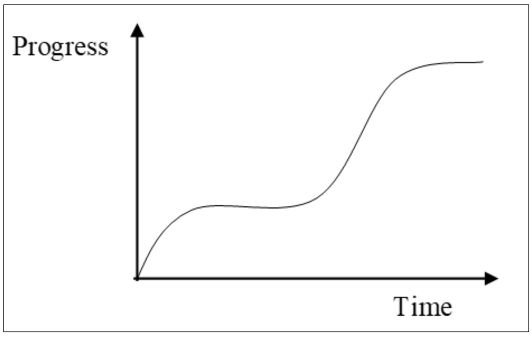


**Figure 6** Learning curve of complex-returns. [2]

### 1.5. The Learning Curve Formula

The general learning formula is given by:

$$y = ax^b$$

Where y = the average rate of progress over the measured duration.

$a$ = the time to complete the task for the first time.
$x$ = the total amount of attempts completed
$b$ = the slope of the function.

The formula can be used as a prediction tool to forecast future performance. [2]

### 1.6. Research Questions

By the end of the study, it was hoped that the following questions would be answered.

- How fast do teacher trainees master statistical concepts?
- How do the way teacher trainees master statistical concepts correspond with the learning curve theory?
- What methods could be used to verify the learning curve theory?
- How would the model be derived?
- What would be the model for the prediction of the percentage of the statistical concepts mastered by the teacher trainees?
- Would the derived model be adequate and appropriate for the prediction of the percentage of the statistical concepts mastered by the teacher trainees?
- How would the adequacy checking of the derived model be done?

## 2. Material and methods

### 2.1. The Experimental Units

The target population of this research was supposed to be all the teacher trainees in all the forty-six public colleges of education in Ghana. However, this was limited to all the level 300 students in all the forty-six public colleges of education because it was the only year group that had all the students offering a statistics related course, Educational Statistics, at the same time in the second semester of the 2020/2021 academic year. According to College of Education Weekly Journal (CoEWJ), the enrollment statistics for level 300 teacher trainees in the 2020/2021 academic year in all the forty-six public colleges of education totaled thirteen thousand and eighty (13,080), which comprised five thousand, one hundred and seventy-six (5,176) females and seven thousand, nine hundred and four (7,904) males. [7].

Also, JOSCO was chosen from the forty-six public colleges of education out of proximity. Here, the level 300 comprised five different classes so far as Educational Statistics lectures were concerned. They were the Science/Mathematics class, Social Studies class, Music/RME class, English/Ghanaian Language class and ICT/Agric class. Through random sampling, Science/Mathematics class, Social Studies class and ICT/Agric class were selected for the study. There were sixty-three (63) students in the Science/Mathematics class, fifty-five (55) students in the ICT/Agric class and seventy-five (75) students in the Social Studies class, totaling one hundred and ninety-three (193) students, which was about 1.476% of 13,080. This served as the experimental units for the study. Out of the 193 students, ninety-five (95) were females and ninety-eight (98) were males.

According to Dornyei (2007), from 1% to 10% of a population understudy gives an adequate sampling fraction. [8]

### 2.2. The Experiment

The 193 teacher trainees were taken through a ten-week period tuition unit by unit until all the seven units entailing the various statistical concepts in Educational Statistics course were exhausted. Each lesson lasted for two hours and each class was engaged three times, making six hours, a week. In effect, each class was engaged sixty hours for the ten-week period, making 180 hours of engagement for the ten-week period for all the three classes.

At the end of every lesson, the teacher trainees would be examined, either through oral or written exercises, and if at least three-quarters (75%) of the 193 teacher trainees were found to have mastered the statistical concepts concerned, it would be assumed that the teacher trainees had mastered the statistical concept taught.

### 2.3. Data Collection

After all the statistical concepts of a unit had been covered and at least 75% of the 193 teacher trainees involved in the study had been found to have mastered the statistical concepts concerned, the percentage of the statistical concepts mastered would be determined by using the relation:

$$y_i = y_{i-1} + \frac{ab_i c x_i}{24 f_i}, \text{ i = 0, 1, 2, …,n.}$$

Where $y_i$= the percentage of the statistical concepts mastered at cumulated time $(t_i)$; $a$ = the number of hours per lesson; $b_i$ = the number of lessons the teacher trainees used to master a statistical concept; $c$ = the total number of units entailed in the course; $f_i$ = the cumulative number of lessons that the teacher trainees would use to master the percentage of the statistical concepts $= f_{i-1} + b_i$ ; and $x_i$ = the percentage of the statistical concepts left to be mastered=100- $y_{i-1}$.

Generally, the various entries of the data set would be as shown in table 1 below. In table 1, the first column, headed by Time ($t_i$)/weeks, deals with the cumulated time (in weeks) that the teacher trainees would use to master cumulated statistical concepts; the second column, headed by Students ($S_i$), deals with the number of teacher trainees who would master the statistical concepts under a particular unit; the third column, headed by Percentage Left ($x_i$), deals with the percentage of statistical concepts left to be mastered by the teacher trainees, and it would be determined by subtracting the previous percentage mastered $(y_{i-1})$ from 100, that is $x_i$ = 100- $y_{i-1}$; and the fourth column, headed by Percentage Mastered $(y_i)$, deals with the cumulated percentage of the statistical concepts that would be mastered by the teacher trainees at cumulated time $(t_i)$.

**Table 1** Layout of tabular representation of data set of percentage of statistical concepts mastered by teacher trainees

| Time (ti)/Weeks | Students (Si) | Percentage Left ($x_i$) | Percentage Mastered($y_i$) |
|---|---|---|---|
| $t_0$ | $S_0$ | $x_0$ | $y_0$ |
| $t_1$ | $S_1$ | $x_1$ | $y_1$ |
| $t_2$ | $S_2$ | $x_2$ | $y_2$ |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| $t_n$ | $S_n$ | $x_n$ | $y_n$ |

### 2.4. Graphical Analysis

According to Spiegel (1972), graph is a pictorial presentation of the relationship between variables. A graph is constructed by plotting set of points on a rectangular co-ordinate system resulting in a physical feature called scatter diagram. [9]

Also, according to Hughes-Hallett and associates (2003, 2nd Edition), under concavity in their book: when a graph of a function is bending upward, it is said to be concave up and the rate of change would be increasing; and when a graph is bending downward, it is said to be concave down and the rate of change would be decreasing. [10]

Following Spiegel's idea, a line chart of smooth curve approximating the data would be used to illustrate the pairs of some entries in the data set, as layout in table 1. This would be done by visualizing and tracing a smooth curved trending line through a scatter diagram, using excel software. The concavity of the graph, hence the rate of change of the function would then be determined for the verification of the learning curve theories.

Generally, the purpose of graphical analysis would be to provide pictorial illustrations for the confirmation of the relationships that would exist between the respective variables and time in the data set, leading to the verification of the learning curve theory.

### 2.5. Concepts Build-Ups

The diminishing-returns learning curve, as stated in reference [2], could laterally mean that the more the learner already knows of a task, the slower he or she learns over time. Mathematically, this could also mean that as the percentage of the task mastered by a learner increase, the rate at which the learner learns decreases over time. However, by Bills' description of the learning curve of increasing progress (in reference [4]), the percentage mastered by the learner would still be increasing over time.

In verification of this, this research was conducted to derive a mathematical model that would be used to evaluate how fast learners (teacher trainees) master a task (statistical concepts). Since the term "how fast" could be mathematically interpreted as "the rate of change", the focus would be on finding an appropriate function whose rate of change would decrease as the function itself increases, hence, finding an appropriate mathematical concept(s) that would be applied in the model derivation process. The researchers then landed on differential equations.

According to Hughes-Hallett and associates (1994), differential equation is an equation that relates one or more functions and their derivatives. They added that, in applications: the functions generally represent physical quantities; the derivatives represent the rate of change of the functions; and differential equation defines the relationship between the two. [11]

Moreover, Chasnov (2001), in his book entitled "Elements of Differential Equations", classified differential equations into ordinary differential equation (ODE) and partial differential equation (PDF). He defined ODE as a differential equation which involves a function and its derivatives, and contains only one independent variable and one or more of its derivatives with respect to the variable. That is:

$$\frac{dy}{dx} = y \text{ ......................(1)}$$

Where x is a single independent variable and y is a dependent variable.

He also defined PDE as a differential equation which involves only partial derivatives of one or more functions of two or more independent variables. That is:

$$\frac{dy}{dx} + \frac{dy}{du} = 0 \text{ .......................(2)}$$

Where x and u are independent variables and y is a dependent variable. [12]

In specification, "how fast teacher trainees master statistical concepts" would transitively represent the "derivative of the mastered statistical concepts", which would be the rate of change of the mastered statistical concept. Here, time would be the only independent variable. Therefore, the ordinary differential equation (ODE) would comparatively be precise and appropriate to be used for the derivation of the model.

### 2.6. Model Formulation

In reference [13], Ross (2007) asserted that: in the process of mathematical model formulation relating to differential equations, certain simplifying assumptions generally have to be made in order that the resulting model would be tractable; if the actual situation in a certain aspect of the problem is of a relatively complicated nature, it is modified by assuming an approximate situation that is comparatively simple in nature which would lead to the elimination of certain relatively unimportant aspects of the problem entirely; and even though these changes would result in a differential equation which would actually be that of an idealized situation, the information that would be obtained from it would be of the greatest value to the researcher.

Following Ross's assertions, '$y$' would represent a percentage of statistical concepts that would be mastered by the teacher trainees and '$t$' would be the time that would be used to master the percentage of statistical concepts. This would imply that '$\frac{dy}{dt}$' would be the rate at which the teacher trainees would master the percentage of statistical concepts. This would mean that $y$ would be a function time ($t$).

With y being a function of $t$, the researchers assumed three different situations that could likely arise, namely:

- teacher trainees who would start at $t$ = 0 and would eventually master 100% of the statistical concepts
- teacher trainees who would start later but would eventually master 100% of the statistical concepts
- teacher trainees who would start at $t$ = 0 but would not master 100% of the statistical concepts

However, in this research, all the teacher trainees were assumed to start from $t$ = 0, even though some of them might have been introduced to certain aspects of the course earlier. Also, due to certain limitations caused by both external and internal factors during the experiment, it would be prudent to assume that the teacher trainees would not master 100% of the statistical concepts they were taught. Hence, situation (iii) would be considered under the research.

By the concept of diminishing-returns learning curve (reference [4]), the rate (how fast) at which the teacher trainees mastered the statistical concepts ($\frac{dy}{dt}$) would decrease as the percentage of statistical concepts mastered ($y$) increased. Also, from Bill's eliminative/declining learning curve (reference [5]), $\frac{dy}{dt}$ would decrease with a decrease in the percentage of statistical concepts left ($x$=100 – y), i.e. the eliminative/declining function. Mathematically, $\frac{dy}{dt}$ would be directly proportional to $x$ (100 – $y$). Assuming the constant of proportionality is one (1), then the rate of change of the statistical concepts mastered by the teacher trainees would be equal to the percentage of statistical concepts left ($x$) to be mastered by the teacher trainees.

An ordinary differential equation would be set up for the relation and then solved to obtain a solution for the differential equation.

### 2.7. Solution of a Differential Equation

According to Thomas (1997), in order to obtain the useful information from a derived differential equation, the differential equation would be solved to obtain a solution of the differential equation. [14]

Also, Adams, Thomas and Hass (1998), highlighted the following information about a solution of differential equation:

A solution of a differential equation is a relation between the independent and the dependent variables which is free of derivatives of any order, and which satisfies the differential equation identically. The solution could be general or particular. A general solution of an nth order differential equation is the one that involves n necessary arbitrary constants. A particular solution of a differential equation is a solution obtained from the general solution by assigning specific values to the arbitrary constants. The conditions for calculating the values of the arbitrary constants can be provided in the form of initial-value problem or boundary conditions, through which initial conditions are provided on the solution that would be needed in the determination of the best solution. The initial-value of the independent variable ($x$) is symbolized $x_0$and that of the dependent variable ($y$) is symbolized$y_0$. [15]

According to Stewart (2011), differential equations can be solved by such methods as series methods, numerical methods, and graphical methods. The series methods yield solutions in the form of infinite series. The numerical methods give approximate values of the solution functions corresponding to selected values of the independent variables. The graphical methods produce approximately the graphs of solutions (i.e. the integral curves). [16]

In effect, numerical method of solving the derived differential equation would be considered for the study.

To make room for errors which could be encored during the experiment, Walpole and associates (2007) suggested that the derived model (function) could be expressed in the natural (untransformed) variables to produce an additive error model. [17]

The error described by Walpole and associates (2007) is technically called residual and it reflects mere random variation or pure experimental errors. Mathematically, it could be determined by finding the difference between the response value and the estimated or predicted value of the response value. The residual is symbolized $E_i$, where $i$ = 1, 2, 3, …, n.

### 2.8. Model Adequacy Checking

The main processes involved in model adequacy checking would be residual analyses. The purpose of the residual analyses was to check whether the errors ($E_i$) are normal and as such would not affect the results of the research in anyway. That would be done by setting up the assumptions that the random variables $E_i = E_1, E_2, E_3, \ldots, E_n$are independent and normally distributed with mean zero (0) and common variance ($\sigma^2$), as highlighted by Milton and Arnold (1995) in reference [18].

Also, Bowerman, O'Connell and Hand (2001), idealized on page 478 of their book entitled "Business Statistics in Practice", that: if the assumptions hold, the residuals should look like they have been randomly and independently selected from normally distributed populations having mean (0) and variance ($\sigma^2$). However, in reality, the assumptions will not hold exactly, as such, it is important to point out that mild departures from the assumptions do not seriously hinder our ability to use a model to make predictions. [19]

Before performing the residual analyses, the estimated or predicted values ($\hat{y}_i$) of the response value ($y_i$) would be calculated using the solution of the derived differential equation and the corresponding residuals ($E_i$) would be computed by using the relation $E_i = y_i - \hat{y}_i$ .

The residual analyses would be performed in two ways – the graphical and the numerical ways of residual analyses. The graphical analyses would be done by plotting the following graphs:

- Residual plot of residual (Ei) on the vertical axis against the independent variable, time ($t_i$), on the horizontal axis.
- Residual plot of residual (Ei) on the vertical axis against the estimated or predicted values ($\hat{y}_i$) on the horizontal axis.
- Normal probability plot of the standardized residuals.

In residual plots (i) and (ii), for the model to be adequate, it would be expected that the points should not exhibit distinct patterns. That is the residuals should be randomly distributed about the zero horizontal line such that all but very few standardized residuals should lie between -3 and +3 (that is all but a few residuals should be within 3 standard deviations of the expected mean value,0). This would mean that the assumption of common variance ($\sigma^2$) would be valid. Hence, the adequacy of the model would be fulfilled.

In the case of the normal probability plot (iii), the residuals ($E_i$) would be arranged in ascending order such that $E_1, E_2, E_3, \ldots, E_n$ are in respective order from smallest ($E_1$) to the biggest ($E_n$). The $E_i$ would then be plotted on the vertical axis against a point called $Z_i$ (standard normal score) on the horizontal axis.

According to Bowerman, O'Connell and Hand (2001), reference [19], $Z_i$ is the point on the horizontal axis under the standard normal curve so that the area under the curve to the left of $Z_i$ is $(3_i - 1)/(3n + 1)$, i = 1, 2, 3, …, n. This implies that the area under the standard normal curve between $Z_i$ and 0 would be equal to $0.5 - [(3_i - 1)/(3n + 1)]$. The corresponding $Z_i$ value would then be read from the normal distribution table.

With the normal probability plot, it would be expected that the coordinate points should exhibit a straight-line appearance to indicate that the normality assumption of the residuals ($E_i$) is valid. Hence, the model would be adequate for the prediction of the dependent variable ($y_i$). On the other hand, if the coordinate points don't exhibit some kind of linearity, then the normality assumption would be violated. [19]

In the case of the numerical residual analyses, Montgomery (2001), in reference [20], recommended the use of the statistic, the standardized residual ($d_i$), given by:

$$d_i = \frac{E_{i*}}{S_E}$$

Where $E_{i*}$ is the largest among all the residuals and $S_E$ is the error (residual) standard deviation, given by:

$$S_E = \sqrt{\frac{\sum {E_i}^2}{N}}$$

In their book entitled "Outliers in Statistical Data", Barnett and Lewis (1994), termed the largest residual ($E_i$) among all the residuals as outlier. The outlier would be detected by considering the largest among the computed residuals and this is one of the procedures outlined by Barnett and Lewis (1994) in their book. It would then be confirmed from the normal probability plot by considering the residual coordinate of the point which appeared to be vertically far from the line-of-best-fit than any of the other coordinate points on the graph. [21].

According to Mongomery (2001), reference [20], if the residuals ($E_i$) are normal with mean zero and unit variance ($\sigma^2$), then the calculated standardized residual ($d_i$) should lie between ±3, inclusive, (that is $-3 \leq d_i \leq 3$). Hence, the model would be adequate for the various predictions.

## 3. Results

### 3.1. The Data Set

With the purpose of deriving a model and evaluating how fast teacher trainees learn statistical concepts, a ten-week period experiment was conducted and the data on the various entries collected.

For the first entries, the initial-value conditions that if the teacher trainees started learning at time ($t_0$) = 0 week, the percentage of statistical concepts mastered ($y_0$) would be $y_0 = 0$, the percentage of statistical concepts left ($x_0$) to be mastered would be $x_0 = 100\%$ and the number of teacher trainees who would master the statistical concepts ($S_0$) would be $S_0 = 0$. The rest of the entries would be determined by using the relation:

$$y_i = y_{i-1} + \frac{ab_icx_i}{24f_i}, i = 0, 1, 2, \ldots, 8. \text{ (with the various terms defined earlier on page 8)}$$

Also, it is important to note that in this study, the number of hours per lesson ($a$ = 2) and the number of units/chapters ($c$) under Educational Statistics course which was $c$ = 7 units/chapters were fixed entities. However, the other terms ($b_i$, $f_i$, and $x_i$) varied over time ($t_i$). The number of lessons per week and per a class was also three (3).

For $y_1$: Two lessons were used and 180 (93.3%) of the teacher trainees mastered the concept of Nature of Statistics. This implies that the time used was $t_1 = \frac{2}{3}$ week.

$$=>b_1 = 2 \text{ lessons;}$$

$$f_1 = f_{1-1} + b_1 = f_0 + b_1 = 0 + 2 = 2;$$

$$x_1 = 100 - y_{1-1} = 100 - y_0 = 100 - 0 = 100\%;$$

$$y_1 = ?$$

$$=>y_1 = y_{1-1} + \frac{ab_1cx_1}{24f_1} = y_0 + \frac{2\times2\times7\times100}{24\times2} = 0 + 58.333 = 58.333\%$$

For $y_2$: Four lessons were used and 163 (84.5%) of the teacher trainees mastered the concept of Data Management and Representation. This implies that the time used was $t_2 = \frac{2}{3} + \frac{4}{3} = 2$ weeks.

$$=>b_2 = 4 \text{ lessons;}$$

$$f_2 = f_{2-1} + b_2 = f_1 + b_2 = 2 + 4 = 6 \text{ lessons;}$$

$$x_2 = 100 - y_{2-1} = 100 - y_1 = 100 - 58.333 = 41.667\%;$$

$$y_2 = ?$$

$$=>y_2 = y_{2-1} + \frac{ab_2cx_2}{24f_2} = y_1 + \frac{2\times4\times7\times41.667}{24\times6} = 58.333 + 16.204 = 74.537\%$$

For $y_3$: Three lessons were used and 160 (82.9%) of the teacher trainees mastered the concept of Measures of Scatter/Dispersion. This implies that the time used was $t_3 = t_2 + \frac{3}{3} = 2 + 1 = 3$ weeks.

$$=>b_3 = 3 \text{ lessons;}$$

$$f_3 = f_{3-1} + b_3 = f_2 + b_3 = 6 + 3 = 9 \text{ lessons;}$$

$$x_3 = 100 - y_{3-1} = 100 - y_2 = 100 - 74.537 = 25.463\%;$$

$$y_3 = ?$$

$$=>y_3 = y_{3-1} + \frac{ab_3cx_3}{24f_3} = y_2 + \frac{2\times3\times7\times25.463}{24\times9} = 74.537 + 4.951 = 79.488\%$$

For $y_4$: Three lessons were used and 165 (85.5%) of the teacher trainees mastered the concept of Measures of Positions. This implies that the time used was $t_4 = t_4 + \frac{3}{3} = 3 + 1 = 4$ weeks.

$$=>b_4 = 3 \text{ lessons;}$$

$$f_4 = f_{4-1} + b_4 = f_3 + b_4 = 9 + 3 = 12 \text{ lessons;}$$

$$x_4 = 100 - y_{4-1} = 100 - y_3 = 100 - 79.488 = 20.512\%;$$

$$y_4 = ?$$

$$=>y_4 = y_{4-1} + \frac{ab_4cx_4}{24f_4} = y_3 + \frac{2\times3\times7\times20.512}{24\times12} = 79.488 + 2.991 = 82.479\%$$

For $y_5$: Five lessons were used and 155 (80.3%) of the teacher trainees mastered the concept of Measures of Relationship. This implies that the time used was $t_5 = t_4 + \frac{5}{3} = 4 + \frac{5}{3} = \frac{17}{3}$ weeks.

$$=>b_5 = 5 \text{ lessons;}$$

$$f_5 = f_{5-1} + b_5 = f_4 + b_5 = 12 + 5 = 17 \text{ lessons;}$$

$$x_5 = 100 - y_{5-1} = 100 - y_4 = 100 - 82.479 = 17.521\%;$$

$$y_5 = ?$$

$$=> y_5 = y_{5-1} + \frac{ab_5cx_5}{24f_5} = y_4 + \frac{2\times5\times7\times17.521}{24\times17} = 82.479 + 3.006 = 85.485\%.$$

For $y_6$: Five lessons were used and 145 (75.1%) of the teacher trainees mastered the concept of Normal Distribution. This implies that the time used was $t_6 = t_5 + \frac{5}{3} = \frac{17}{3} + \frac{5}{3} = \frac{22}{3}$ weeks.

$$=> b_6 = 5 \text{ lessons;}$$

$$f_6 = f_{6-1} + b_6 = f_5 + b_6 = 17 + 5 = 22 \text{ lessons;}$$

$$x_6 = 100 - y_{6-1} = 100 - y_5 = 100 - 85.485 = 14.515\%;$$

$$y_6 = ?$$

$$=> y_6 = y_{6-1} + \frac{ab_6cx_6}{24f_6} = y_5 + \frac{2\times5\times7\times14.515}{24\times22} = 85.485 + 1.924 = 87.409\%.$$

For $y_7$: Five lessons were used and 149 (77.2%) of the teacher trainees mastered the concept of Introduction to Hypothesis Testing. This implies that the time used was $t_7 = t_6 + \frac{5}{3} = \frac{22}{3} + \frac{5}{3} = 9$ weeks.

$$=> b_7 = 5 \text{ lessons;}$$

$$f_7 = f_{7-1} + b_7 = f_6 + b_7 = 22 + 5 = 27 \text{ lessons;}$$

$$x_7 = 100 - y_{7-1} = 100 - y_6 = 100 - 87.409 = 12.591\%;$$

$$y_7 = ?$$

$$=> y_7 = y_{7-1} + \frac{ab_7cx_7}{24f_7} = y_6 + \frac{2\times5\times7\times12.591}{24\times27} = 87.409 + 1.360 = 88.769\%.$$

For $y_8$: Three lessons were used and 161 (83.4%) of the teacher trainees mastered the concept of Simple Regression. This implies that the time used was $t_8 = t_7 + \frac{3}{3} = 9 + 1 = 10$ weeks.

$$=> b_8 = 3 \text{ lessons;}$$

$$f_8 = f_{8-1} + b_8 = f_7 + b_8 = 27 + 3 = 30 \text{ lessons;}$$

$$x_8 = 100 - y_{8-1} = 100 - y_7 = 100 - 88.769 = 11.231\%;$$

$$y_8 = ?$$

$$=> y_8 = y_{8-1} + \frac{ab_8cx_8}{24f_8} = y_7 + \frac{2\times3\times7\times11.231}{24\times30} = 88.769 + 0.655 = 89.424\%.$$

The summary of the data set collected is as shown in table 2 below. The first column, headed by time ($t_i$)/weeks, deals with the total time, in weeks, it took the teacher trainees to master an accumulated percentage of statistical concepts; the second column, headed by students ($S_i$), deals with the number of teacher trainees (out of 193) who had mastered the statistical concepts under a unit; the third column, headed by percentage left ($x_i$), deals with the percentage of the course outline yet to be mastered by the teacher trainees by the time($t_i$). It was gotten by subtracting the previously mastered percentage concepts ($y_{i-1}$) from 100% (i.e. $x_i = 100 - y_{i-1}$); and the fourth column, headed by percentage mastered ($y_i$), deals with the accumulated percentage of statistical concepts mastered by the teacher trainees as at time ($t_i$), in weeks.

**Table 2** Approximate of percentage of statistical concepts mastered by teacher trainees at time ($t_i$), in weeks

| **Time ($t_i$)/Weeks** | **Students ($S_i$)** | **Percentage left ($x_i$)/%** | **percentage mastered ($y_i$)/%** |
|---|---|---|---|
| 0 | 0 | 100 | 0 |
| 0.7 | 180 | 100 | 58.3 |
| 2 | 163 | 41.7 | 74.5 |
| 3 | 160 | 25.5 | 79.5 |
| 4 | 165 | 20.5 | 82.5 |
| 5.7 | 155 | 17.5 | 85.5 |
| 7.3 | 145 | 14.5 | 87.4 |
| 9 | 149 | 12.6 | 88.8 |
| 10 | 161 | 11.2 | 89.4 |

From table 2 above, the number of teacher trainees who mastered the statistical concepts of a subject matter under a particular unit varied with the highest being 180 (93.3%) out of the 193 under the concept of Nature of Statistics and the lowest number being 145 (75.1%) out of the 193 under the concept of Normal Distribution. However, the percentage of statistical concepts yet to be mastered by the teacher trainees kept on decreasing from 100% in the first week to 11.2% by the end of the tenth week. On the other hand, the percentage of the statistical concepts mastered by the teacher trainees kept on increasing from 58.3% in the first week to 89.4% in the tenth week.

### 3.2. Tabular Analysis of the Data

Now finding the rate at which the teacher trainees mastered the statistical concepts right from the time ($t_1$) to the time ($t_8$), using the relation:

$$\frac{dy}{dt} = \frac{y_i - y_{i-1}}{t_i - t_{i-1}}$$

Where $\frac{dy}{dt}$ is the rate (how fast) the teacher trainees mastered the statistical concepts over time, in weeks. Table 3 below gives the summary of the results.

**Table 3** The rate at which the teacher trainees mastered the statistical concepts within the ten-week period of the experiment

| $t_i$**(weeks)** | $y_i$**(%)** | $\frac{dy}{dt}$**(% per week)** |
|---|---|---|
| 0.7 | 58.3 | 87.5 |
| 2 | 74.5 | 12.2 |
| 3 | 79.5 | 5.0 |
| 4 | 82.5 | 3.0 |
| 5.7 | 85.5 | 1.8 |
| 7.3 | 87.4 | 1.2 |
| 9 | 88.8 | 0.8 |
| 10 | 89.4 | 0.7 |

In table 3, the first column, headed by $t_i$(where i = 1, 2, 3, ...,8) deals with the accumulated time the teacher trainees used to master the statistical concepts; the second column, headed by $y_i$ (where i = 1, 2, 3, ...,8) deals with the percentage of the statistical concepts mastered by the teacher trainees as at time $(t_i)$; and the third column, headed by $\frac{dy}{dt}$, deals with the rate at which the teacher trainees mastered the statistical concepts per week within the ten-week period of the experiment.

From table 3, the rate at which the teacher trainees mastered the statistical concepts $(\frac{dy}{dt})$ started on a high note from 87.5% per week and kept on decreasing as the weeks went by to as low as 0.7% per week by the end of the tenth week of the experiment. However, the percentage of the statistical concepts mastered by the teacher trainees kept on increasing right from 58.3% through the weeks to 89.4% by the end of week ten.

### 3.3. Graphical Analysis of the Data

For the purpose of confirming the relationships that existed between the various variables in the data set of table 2, graphs of line chart of approximating smooth curves from excel software were used to illustrate the data of the various pairs of variables as follows:

#### *3.3.1. Percentage left $(\boldsymbol{x_i})$ against Time $(\boldsymbol{t_i})$*

With this, the percentage of statistical concepts left $(x_i)$ to be mastered by the teacher trainees was scaled on the vertical axis against the time $(t_i)$ used by the teacher trainees to master the various statistical concepts, as shown in figure 7 below:

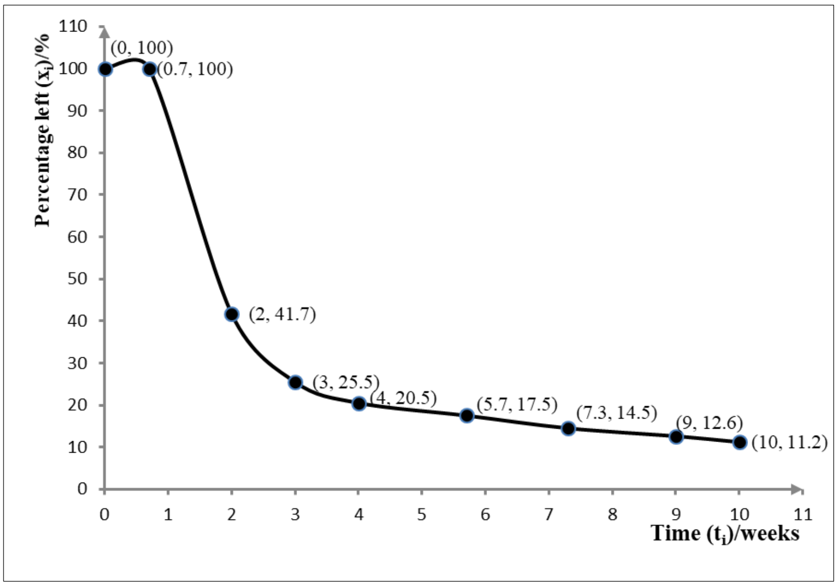


**Figure 7** Excel output of graph illustrating the percentage of statistical concept left to be mastered by teacher trainees against the time they used to master the various statistical concepts

From the graph (figure 7), the trending line started on a flat note between the points (0,100) and (0.7,100), descended steeply from point (0.7,100) to the point (3,25.5) and then from there began to descend gently to the end point (10,11.2). However, the trending line approaches the horizontal axis asymptotically. In all, the graph is concave up since the trending line bends upward slightly from left to right.

#### *3.3.2. Percentage Mastered $(\boldsymbol{y_i})$ against Time $(\boldsymbol{t_i})$*

Plotting the graph of percentage mastered $(y_i)$against time$(t_i)$, the cumulated percentage of statistical concepts mastered $(y_i)$ by the teacher trainees was scaled on the vertical axis whiles the cumulated time$(t_i)$ used to master the statistical concepts was scaled on the horizontal axis. This is as shown in figure 8 below:

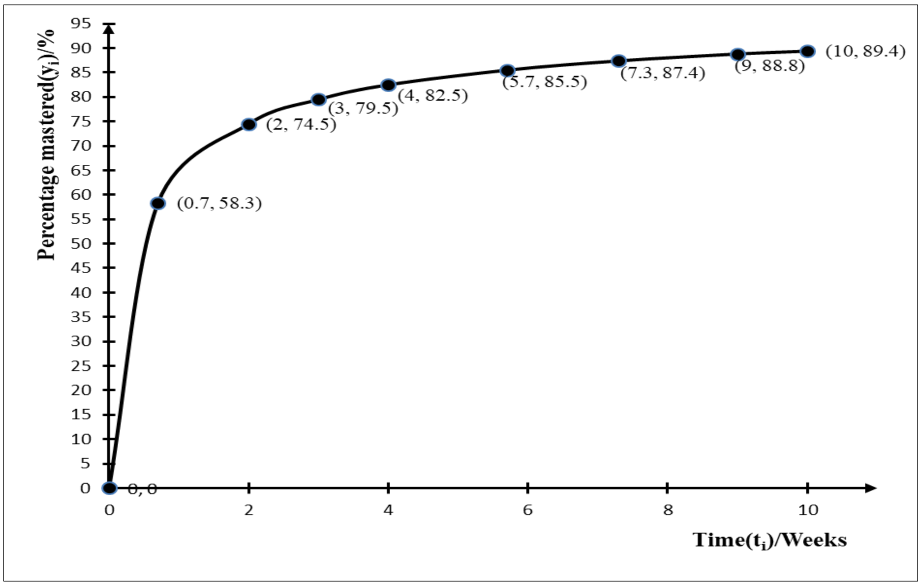


**Figure 8** Excel output of graph of percentage of statistical concepts mastered by teacher trainees against the cumulated time they used to master the concepts

In figure 8, the approximating smooth curve of the graph ascended steeply from the point (0,0) rightwards to the point (0.7,58.3) and then ascended gently to the end point (10,89.4). However, the graph approaches the imaginary $y_i$ = 100% horizontal line asymptotically. In all, the graph is concave down from left to right.

Also, graphs of line chart of approximating smooth curves were used to illustrate the data in table 3 in order to confirm the relationships between the variables. The graphs are as follows:

### *3.3.3. Rate ($\frac{dy}{dt}$) against Percentage Mastered ($y_i$)*

Plotting a graph of rate ($\frac{dy}{dt}$), in percentage per week, at which the teacher trainees mastered the statistical concepts on the vertical axis against the cumulated percentage of statistical concepts mastered ($y_i$) by the teacher trainees on the horizontal axis, the detail is as shown in figure 9 below:

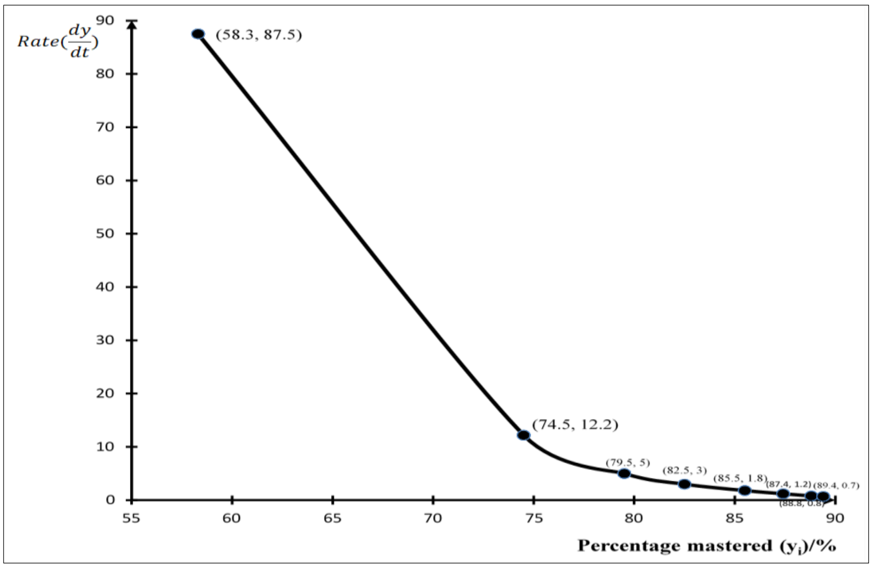


**Figure 9** Excel output of graph of rate at which teacher trainees mastered statistical concepts against the percentage of cumulated statistical concepts they mastered

From the graph in figure 9, the trending line is descending steeply from left to right between the points (58.3,87.5) and (74.5,12.2), and then turn to descend gently to the end at point (89.4,0.7).

### *3.3.4. Rate ($\frac{dy}{dt}$) against Percentage Left ($x_i$)*

Plotting a graph of rate ($\frac{dy}{dt}$), in percentage per week, at which the teacher trainees mastered the statistical concepts on the vertical axis against the percentage of statistical concepts left to be mastered ($x_i$) by the teacher trainees on the horizontal axis, the detail is as shown in figure 10 below:

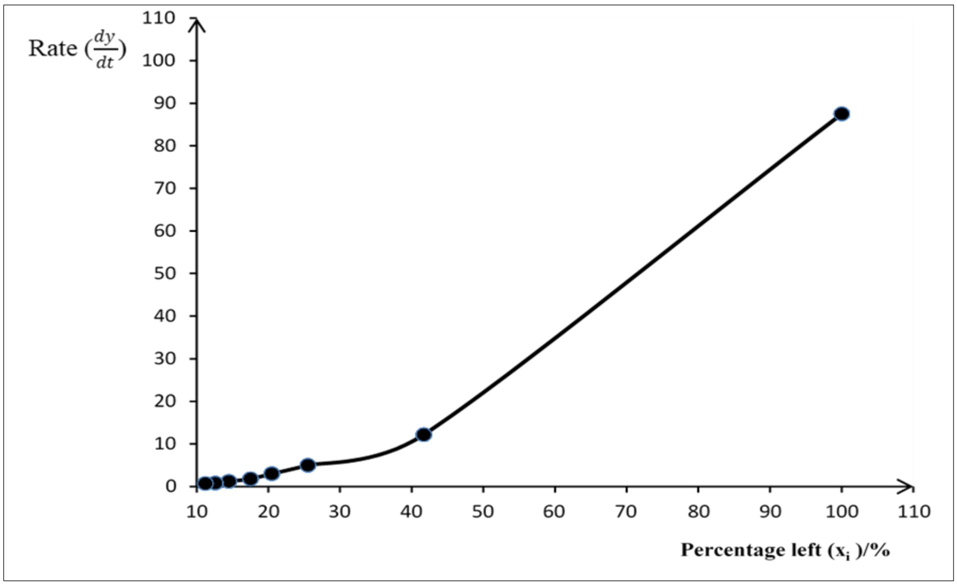


**Figure 10** Excel output of graph of rate at which teacher trainees mastered statistical concepts against the percentage of statistical concepts left to be mastered

From the graph in figure 10, the trending line is generally in ascendency from left to right. However, in the early stages, the ascendency was gentle and grew steeper towards the latter stages.

## 3.4. Model Formulation

It could be deduced from figure 10 that the rate ($\frac{dy}{dt}$) at which the teacher trainees mastered the statistical concepts is directly proportional to the percentage of statistical concepts left ($x$) to be mastered by the teacher trainees at time ($t$), in weeks. This is confirmed by the corresponding values of $\frac{dy}{dt}$ in table 3 to that of $x$ in table 2, since a decreased in $x$ transitively caused a decreased in $\frac{dy}{dt}$. Thus, mathematically:

$$\frac{dy}{dt} \propto x$$

But$x = (100 - y)\%$. Also, let's assumed that the constant of proportionality is equal to one (1).

Hence, a differential equation to model how fast ($\frac{dy}{dt}$) the teacher trainees mastered the statistical concepts in time (t), in weeks, would be:

$$\frac{dy}{dt} = x = 100 - y \text{ ....................................} (1)$$

This differential equation (1) would be expected to give the required information about the unknown percentage of the statistical concepts mastered ($y$) by the teacher trainees within the time ($t$), in weeks.

To find the solution of the differential equation (1), both sides are first negated because the rate ($\frac{dy}{dt}$) decreased and the percentage of the statistical concepts left ($x = 100 - y$) to be mastered also decreased with time ($t$), in weeks. Thus the differential equation (1) would now be given by:

$$-\left(\frac{dy}{dt}\right) = -(100 - y)$$

$$=> \quad -\left(\frac{dy}{dt}\right) = y - 100$$

Separating the variables,

$$=> \left(\frac{1}{y - 100}\right) dy = -dt$$

Integrating both sides,

$$=> \int \left(\frac{1}{y - 100}\right) dy = -\int dt$$

=> $\ln|y - 100| = -t + A$, where $A$ is constant.

Taking the anti-logarithm of both sides,

$$=> |y - 100| = e^{-t+A} = e^{-t} \times e^{A} = e^{A} \times e^{-t}$$

Let $B = e^{A}$ (Constant).

$$=> |y - 100| = Be^{-t}$$

$$=> y - 100 = \pm Be^{-t}$$

Let $C = \pm B$ (Constant).

$$=> y - 100 = Ce^{-t}$$

$$=> y = 100 + Ce^{-t} \text{..........................(2)},$$

Where$C$ is an arbitrary constant.

To verify whether equation (2) identically satisfied the differential equation (1), substitute equation (2) into the differential equation (1). Thus:

i. On the left side of the equal sign (=);

$$\frac{dy}{dt} = \frac{d}{dt}(100 + Ce^{-t}) = 0 + (-Ce^{-t}) = -Ce^{-t} \text{..........................(3)}$$

ii. On the right side of the equal sign (=);

$$100 - y = 100 - (100 + Ce^{-t}) = 100 - 100 - Ce^{-t} = 0 - Ce^{-t} = -Ce^{-t} \text{............(4)}$$

Since (3) = (4), the results for the left side and the right side of the differential equation (1) are identical. Hence, equation (2) satisfies the differential equation (1) identically.

Therefore, the general solution of the differential equation (1) is:

$$y = 100 + Ce^{-t}$$

Now to find the arbitrary constant ($C$), the initial-value conditions are set in correspondence with the assumption that at time ($t$) = 0 week, all the teacher trainees had not mastered any statistical concept, making $y$ initially zero (i.e. $y = 0$).

So substituting $t=0$ and $y=0$ into equation (2),

$$=> \quad 0 = 100 + Ce^{-0}$$

$$=> \quad 0 = 100 + Ce^{0}$$

$$=> \quad 0 = 100 + C$$

$$=> \quad C = -100$$

Substituting $C = -100$ into equation (2),

$$=> y = 100 - 100e^{-t} \text{……………………}(5)$$

By substitution, equation (5) also satisfies the differential equation (1) and the initial-value conditions y = 0 at t = 0. Therefore, equation (5) is the particular solution of the differential equation (1).

Now to make room for errors (residuals) that might have incurred during the experiment, equation (5) is expressed in the natural (untransformed) variables by adding the residuals ($E_i$) to produce the model:

$$y_i = 100 - 100e^{-t_i} + E_i \text{……………………}(6)$$

### 3.5. Model Adequacy Checking

*3.5.1. Residuals*

To conduct the various residual analysis with the purpose of checking whether or not the underlined assumptions that the errors (residuals) which might have been committed during the experiment were random, independent and normally distributed with mean zero (0) and common variance ($\sigma^2$) hold, the estimated or predicted values ($\hat{y}_i$) of the cumulated percentage of the statistical concepts mastered ($y_i$) by the teacher trainees were calculated using the equation:

$$\hat{y}_i = 100 - 100e^{-t_i}, \qquad i = 1, 2, \ldots, 8.$$

Where $e$ is the natural logarithm of 2 and it is of approximate constant value 2.718282. Hence, the corresponding residuals ($E_i$) were calculated using the equation:

$$E_i = y_i - \hat{y}_i, \qquad i = 1, 2, \ldots, 8.$$

The results are as summarized in table 4 below.

In table 4, the first column is headed by serial number (i = 1, 2, 3, …, 8); the second column is headed by the cumulated time ($t_i$), in weeks, taken by the teacher trainees to master the corresponding cumulated percentage of the statistical concepts; the third column is headed by the response value ($y_i$) of the cumulated percentage of the statistical concepts mastered by the teacher trainees; the fourth column is headed by the predicted value ($\hat{y}_i$) of $y_i$; and the fifth column, headed by Residual ($E_i$), is the calculated corresponding residual values.

From table 4, the residual values ($E_i$) ranged between -15.5 to 9.6 with four of them being negative and five being positive. However, the highest value in absolute was 15.5 and it occurred at time ($t_4$) = 4 weeks, while the lowest in absolute was 0.0 and it occurred at time ($t_1$) = 0.0 week.

**Table 4** Residuals ($\boldsymbol{E_i}$) of the cumulated percentage of the statistical concepts mastered by the teacher trainees

| i | Time $(t_i)$/ weeks | Response value $(y_i)$/% | Predicted value $(\hat{y}_i)$/% | Residual $(E_i = y_i - \hat{y}_i)$ |
|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | 0.7 | 58.3 | 48.7 | 9.6 |
| 3 | 2.0 | 74.5 | 65.6 | 8.9 |
| 4 | 3.0 | 79.5 | 95.0 | -15.5 |
| 5 | 4.0 | 82.5 | 72.8 | 9.7 |
| 6 | 5.7 | 85.5 | 99.7 | -14.2 |
| 7 | 7.3 | 87.4 | 81.2 | 6.2 |
| 8 | 9.0 | 88.8 | 99.988 | -11.2 |
| 9 | 10.0 | 89.4 | 99.995 | -10.6 |

*3.5.2. Graphical Residual Analysis*

Performing the graphical residual analysis, using excels software, the following headings were considered:

Residual ($\boldsymbol{E_i}$) against Percentage Left ($\boldsymbol{x_i}$)

Plotting the residuals ($E_i$) on the vertical axis against the percentage of statistical concepts left ($x_i$) - the independent variable - to be mastered by the teacher trainees, the excel output of the graph in figure 11 below was obtained.

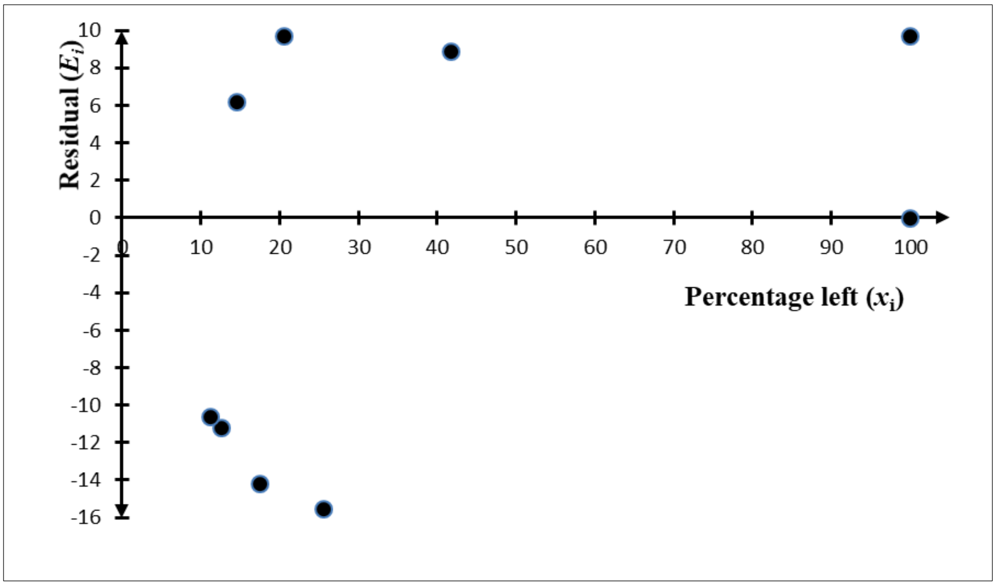


**Figure 11** Excel output of graph of residuals against the percentage of statistical concepts left (independent variable) to be mastered by the teacher trainees

In figure 11 above, the coordinate points have four of them located below and four located above the horizontal axis, and one lying exactly on the horizontal axis. The horizontal axis (0) serves as the mean of the residuals ($E_i$). Also, the coordinate points appear not to exhibit any distinct pattern and as such could be said to be randomly distributed about the horizontal axis (0).

Residual ($\boldsymbol{E_i}$) against Estimated Percentage Mastered ($\boldsymbol{\hat{y}_i}$)

Plotting the residuals ($E_i$) on the vertical axis against the percentage of the estimated statistical concepts mastered ($\hat{y}_i$) by the teacher trainees, the excel output is as shown in figure 12 below:

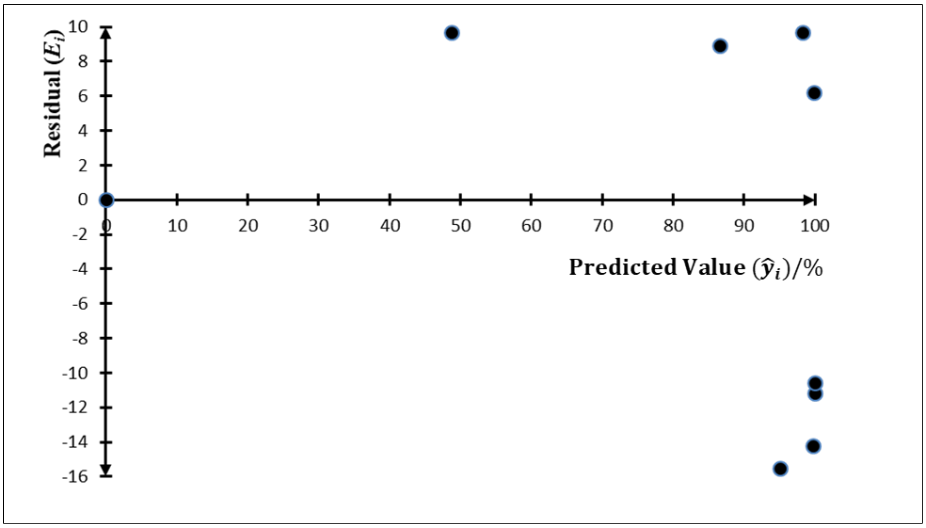


**Figure 12** Excel output of graph of residuals against predicted percentage of the statistical concepts mastered by the teacher trainees

In figure 12, the coordinate points have four of them located below and four located above the horizontal axis, and one lying exactly on the vertical and the horizontal axes intercept. However, most of the coordinate points appear to be located at the extreme right ends of the horizontal axis, with exhibition of no distinct pattern. Therefore, they could be said to be randomly distributed about the horizontal axis (0).

The Normal Probability Plot

With this, the residual values were first arranged in ascending order in correspondence with the respective Z-scores ($Z_i$) of the estimated percentage of statistical concepts mastered ($\hat{y}_i$), as in table 5 below. The Z-scores ($Z_i$) were obtained by using the excel software.

**Table 5** Residuals in ascending order corresponding to the respective Z-scores of the estimated percentage of statistical concepts mastered

| **Ordered Residuals ($E_i$)** | **estimated statistical concepts mastered ($\hat{y}_i$)/%** | **Z-scores ($Z_i$)** |
|---|---|---|
| -15.5 | 95.0 | -2.34 |
| -14.2 | 99.7 | -0.932 |
| -11.2 | 99.988 | 0.162 |
| -10.6 | 99.995 | 0.409 |
| 0.0 | 0.0 | 0.500 |
| 6.2 | 81.2 | 0.543 |
| 8.9 | 65.6 | 0.551 |
| 9.65 | 48.7 | 0.552 |
| 9.7 | 72.8 | 0.553 |

Now plotting the ordered residuals ($E_i$) on the vertical axis against the Z-scores ($Z_i$), of the estimated percentage of statistical concepts mastered by the teacher trainees, on the horizontal axis, the excel output is as shown in figure 13 below:

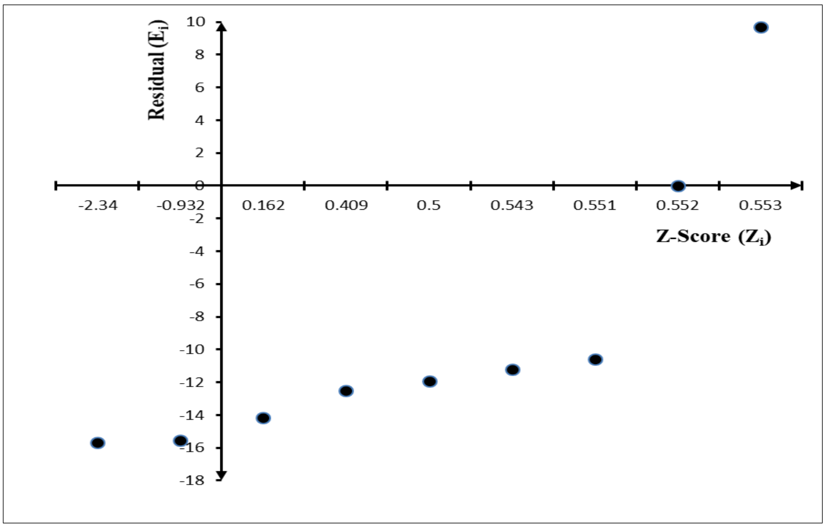


**Figure 13** Excel output of normal probability plot of the standardized residuals

In figure 13, one coordinate point lies above the horizontal axis, one lies exactly on the horizontal axis while the rest lie below the horizontal axis. Also, it can be visualized that those below the horizontal axis portray some kind of linearity (i.e. exhibition of straight line appearance).

*3.5.3. Numerical Residual Analysis*

In the conduct of the numerical residual analysis, with reference to table 4, the largest among the residuals ($E_{i*}$) was found to be $E_{4*} = -15.5$ and the residual standard deviation ($S_E$) was calculated by using the function:

$$S_E = \sqrt{\frac{\sum(E_i)^2}{N}} = \sqrt{\frac{(0.0)^2 + (9.6)^2 + (8.9)^2 + \cdots + (-10.6)^2}{9}}$$

$$= \sqrt{\frac{1317.706}{9}}$$

$$= \sqrt{146.412}$$

$$= 12.100$$

Therefore, the standardized residual ($d_4$)$= \frac{E_{4*}}{S_E} = \frac{-15.5}{12.100} = -1.28$

The calculated standardized residual ($d_4$)$= -1.28$ lies between – 3 and +3 [i.e. $-3 \leq (d_4 = -1.28) \leq 3$].

## 4. Discussion

From table 2, the number of teacher trainees who mastered the statistical concepts at a particular point in time varied. Also, there was no time where the whole sample size (193) mastered a statistical concept. However, the minimum among the number of teacher trainees who mastered the statistical concepts at a point in time was 145 and this number was about 75.13% of the sample size (193), which was in conformity with the researchers' assumption that at least 75% of the sample size (193) was needed to determine that the teacher trainees had mastered a statistical concept. Not all, this 145 was about 1.109% of the target population (i.e. the 13,080 level 300 teacher trainees in all the 46 public colleges of education in Ghana for the 2020/2021 academic year). Moreover, the 193 sample size was about 1.476% of the target population (13,080). In all, the 1.109% and the 1.476% were adequate sample fractions of the target population, and this is in fulfillment of Donyei's assertion that 1% to 10% of a population understudy gives adequate sample fraction, as stated in reference [8].

Again, in table 2, it was clear that as the weeks went by, the statistical concepts mastered by the teacher trainees increased as a result of a reduction in the percentage of the statistical concepts left to be mastered. Also, the teacher trainees started at time, $t = 0$ week and were not able to master 100% (but rather mastered 89.424%) by the end of the 10th week, which fulfilled the researchers' assumption in situation (iii), as set on page 11 of this report.

From figure 7, the steepness of the descending trending line initially means that there was a drastic decrease in the percentage of the statistical concepts left at the initial stages of the instruction, and the gentle sloping of the trending line towards the horizontal axis indicates that the decreasing of the percentage of the statistical concepts left became gradual towards the end of the experiment. This is in line with Bills' description of the learning curve as eliminative/declining in progress (illustrated in figure 2) under reference [4].

In figure 8, the steeper ascendency of the trending line initially and then becoming gentle towards the end means that the percentage of the statistical concepts mastered by the teacher trainees increased rapidly at the beginning and then turned to increase gradually as the weeks went by. This further implies that the rate at which the teacher trainees mastered the statistical concepts was initially higher but started reducing as the time (in weeks) passed by and even became very low in the end, as confirmed by the concave down nature of the graph in agreement with Hughes-Hallett and associates assertion under concavity, as in reference [10]. This portrays the characteristics of the learning curve of diminishing-returns and Bills' learning curve of increasing in progress, as in references [2] and [4] respectively. Not all, the asymptotic behavior of the trending line towards the imaginary $y_i$=100% horizontal line means that the teacher trainees could not master 100% of the statistical concepts at the end of the 10-week experimental period, and this confirms the validity of the researchers' assumption in situation (iii) on page 11.

From figure 9, the steep descending of the trending line initially and the gentle descending of the line later means that the rate of change ($\frac{dy}{dt}$) of the percentage of the statistical concepts mastered by the teacher trainees was decreasing faster initially but slowed down later as the percentage of the statistical concepts mastered ($y_i$) increased.

Generally, the statistical concepts mastered by the teacher trainees increased as the weeks passed by, just that the students' rate of mastering the statistical concepts was fast and rapid during the initial stages but decreased as the weeks proceeded. This is in line with the general learning curve theory, as stated in reference [2].

However, the decreasing in the teacher trainees' rate of mastering the statistical concepts signaled that may be the teacher trainees had reached their ability limit or their mental transition was occurring or they had lost motivation or they were bored or they were fatigued or they were discouraged by the memorization of the numerous statistical formulas. These are some of the impact factors on the end-results of a learning process, as given in reference [2].

In the case of the graphical residual analyses, the coordinate points in figures 11 and 12 do not exhibit any distinct patterns on each graph and as such could be said to be randomly distributed about the horizontal axis (0). This means that the errors/residuals could be assumed to be independent and randomly distributed with mean (0) and common variance ($\sigma^2$), as stated by Milton and Arnold (1995) in reference [18]. Even though there were mild departures from the assumptions made since the distribution of the points was not exactly uniform about the horizontal axis, this was not too serious to affect the normality assumption of the errors/residuals. Hence, this would not seriously hinder the adequacy of the derived model, as in line with the views of Bowerman, O'Connell and Hand in reference [19].

For the normal probability plot (figure 13), since most of the coordinate points exhibited some kind of linearity trend and only two were outliers, the normality assumption of the errors/residuals was valid. Hence, the derived model was adequate for the estimation of the percentage of the statistical concepts mastered by the teacher trainees. This is also in line with Bowerman, O'Connell and Hand's ideas, as in reference [19]

In the case of the numerical residual analysis, since the calculated standardized residual ($d_4 = -1.28$) falls between $-3$ and +3, inclusive, the errors/residuals were assumed to be normal with mean (0) and common variance (146.412). Hence, the derived model was adequate for the estimation of the percentage of the statistical concepts mastered by the teacher trainees, and this is in conformity with Mongomery's concept about numerical residual analysis in reference [20].

As all the residual analyses (both the graphical and the numerical) showed that the errors/residuals were normal and independent, and therefore the errors/residuals effects were negligible, the model $y_i = 100 - 100e^{-t_i} + E_i$ would then become$y_i = 100 - 100e^{-t_i}$. This model is an exponential function with base $e$ (natural logarithm of 2, i.e. ln2) and exponent – $t$. The negative ($-$) aspect of the time ($t$) indicates that the rate at which the teacher trainees mastered the statistical concepts would decrease even though the percentage of the statistical concepts mastered by the teacher trainees would be on the increase over time (in weeks). Hence, the general learning curve theory that "a learner's proficiency in a task improves over time the more the learner performs the task, as stated in reference [2], is verified.

## 5. Conclusion

The ten-week period used for the experiment was adequate and the data collected for the study was accurate. This is because the sample size and the minimum number of the teacher trainees (among the others) who mastered the statistical concepts at a point in time were adequate sample fractions and true representative of the target population (that is the total enrolment of all the level 300 teacher trainees in the forty-six colleges of education in Ghana for the 2020/2021 academic year.

The results and findings from the tabular and graphical analyses showed that the rate at which the teacher trainees mastered the statistical concepts was initially higher and then decreased gradually as the time (in weeks) passed on, and this proves Bills' learning curve theory of increasing in progress and learning curve theory of diminishing-returns but in contradiction with Wright's experience learning curve theory.

Even though there was reduction in the rate at which the teacher trainees mastered the statistical concepts over time, learning was still in progress and the students' proficiency in the statistical concepts was improving over time as teaching instruction continued, hence the general learning curve theory was verified. However, it was detected that the reduction in the teacher trainees' rate of mastering over time was due to boredom, fatigue and inability to memorize the various statistical formulas on the part of the students.

Ordinary Differential Equation (ODE) was successfully used to derive the model for the estimation of the percentage of the statistical concepts mastered by the teacher trainees. This was done under some assumptions made by the researchers, as suggested by some authors.

The various residual analyses conducted for the model adequacy checking showed that the errors/residuals which might have been committed during the study were normal and independent. This indicates that the various assumptions made by the researchers were valid. Therefore, there was much evidence to neglect the residuals effect ($E_i$) in the derived model ($y_i = 100 - 100e^{-t_i} + E_i$). Hence, the model becomes:

$$y_i = 100 - 100e^{-t_i}$$

and it is adequate and appropriate for the estimation of the percentage of the statistical concepts mastered by the teacher trainees at any point in time ($t$).

This research is unique in the sense that it is the first time a research is being conducted to determine and model 'how fast' teacher trainees master statistical concepts in Ghana

## Compliance with ethical standards

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### *Disclosure of conflict of interest*

No conflict of interest.

### *Statement of informed consent*

Informed consent was obtained from all individual participants included in the study.

## References


[1] Gordor BK, Howard NK. Elements of Statistical Analysis. Accra, Ghana: The City Printers; 2000.

[2] Valamis. Learning Curve Theory: The Definitive Guide [Internet]. Finland: Valamis; © 2021 [cited 2021 June 15]. www.valamis.com

[3] Swift EJ. Studies in Psychology and Physiology of Learning. American Journal of Psychology. 1903. 14(2): 201-251. (https://en.m.wikipedia.org)

[4] Bills AG. General Experiment Psychology. Finland: Valamis; © 1934. (www.valamis.com)

[5] Wright TP. Factors Affecting the Cost of Airplanes. Journal of the Aeronautical Sciences. 1936. 3(4): 122-128. (en.m.wikipedia.org).

[6] https://360learning.com>guide.

[7] https://t-tel.org

[8] Donyei Z. Research Methods in Applied Linguistics: Quantitative, Qualitative and Mixed Methodologies. New York: Oxford University Press; 2007.

[9] Spiegel MR. Theory and Problems of Statistics. SI (Metric) Edition. Singapore: McGraw-Hill Book Company; 1972.

[10] Hughes-Hallett D, et al. Applied Calculus. 2nd Edition. New York: John Wiley and Sons, Inc; 2003.

[11] Hughes-Hallett D, et al. Calculus. Toronto, Canada: John Wiley and Sons; 1994.

[12] Chasnov JR. Elements of Differential Equations. Kowloon, Hong Kong: Clear Water Bay; 2001.

[13] Ross SL. Differential Equations. 3rd Edition. New Delhi, India: John Wiley and Sons, Inc; 2007.

[14] Thomas C. Introduction to Differential Calculus. Sydney, Australia: University of Sydney Press; 1997.

[15] Adams C, Thomas A, Hass J. How to Ace Calculus: The Streetwise Guide. New York, USA: W. H. Freeman and Company; 1998.

[16] Stewart J. Calculus: Early Transcendental. 7th Edition. Toronto, Canada: Brooks/Cole Sengage Learning; 2011.

[17] Walpole RE, Myers HR, Myers SL, Ye K. Probability and Statistics for Engineers and Scientists. 8th Edition. New Jersey: Pearson Education, Inc; 2007.

[18] Milton JS, Arnold JC. Introduction to Probability and Statistics: Principles and Applications for Engineering and Computing Sciences. 3rd Edition. Singapore: McGraw-Hill, Inc; 1995.

[19] Bowerman BL, O'Connell RT, Hand ML. Business Statistics in Practice. 2nd Edition. New York, USA: McGraw-Hill Companies, Inc; 2001.

[20] Montgomery DC. Design and Analysis of Experiment. 5th Edition. New York, USA: John Wiley and Sons, Inc; 2001.

[21] Barnett V, Lewis T. Outliers in Statistical Data. 3rd Edition. New York: Wiley and Sons; 1994.