Open Access Research Journal of
Engineering and Technology
Journals home page: https://oarjpublication/journals/oarjet/
ISSN: 2783-0128 (Online)

OARJ OPEN ACCESS RESEARCH JOURNALS

(RESEARCH ARTICLE)

Modeling momentum transfer effect in deoxygenation for sustainable marine management and pollution control in natural gas wastewater treatment

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Open Access Research Journal of Engineering and Technology, 2025, 08(01), 079-094

Publication history: Received on 29 January 2025; revised on 17 March 2025; accepted on 20 March 2025

Article DOI: https://doi.org/10.53022/oarjet.2025.8.1.0035

Abstract

This study investigates and models the impact of momentum transfer in Deoxygenation for Sustainable Marine Management and Pollution Control in Natural Gas Wastewater Treatment, on the concentrations of dissolved oxygen (DO) and biochemical oxygen demand (BOD) within produced natural gas wastewater in a pond treatment system, particularly under conditions of high precipitation. A comprehensive mathematical model was crafted to quantify the effects of momentum transfer on DO and BOD levels. The fluctuations in BOD and DO concentrations were linked to the influences of momentum transfer, leading to the development and simulation of a mathematical framework that elucidates the role of momentum transfer in the bioremediation of wastewater. The separation of variables technique was employed to derive the general solution associated with momentum transfer. This general solution was subsequently correlated with the specifically developed models for BOD and DO to yield a final equation, which was then simulated across various parameters. The findings from this theoretical model were rigorously compared with empirical data, demonstrating a strong correlation that underscores the utility of the developed model in both monitoring and forecasting the implications of momentum transfer on BOD and DO concentrations. This is particularly relevant for continuous discharges of produced natural gas wastewater into pond systems under high precipitation scenarios. The results highlight the critical function of momentum transfer in enhancing the efficiency of wastewater treatment processes, especially concerning regular effluent discharges stemming from natural gas production operations. This research offers a valuable analytical framework aimed at optimizing wastewater management practices in regions experiencing significant rainfall, thereby promoting sustainable environmental practices in the natural gas sector.

Keywords: Momentum; Natural Gas Processing; Deoxygenation; Marine Pollution Control

1. Introduction

Natural gas production and processing, whether it is from associated or non-associated sources, generates a complex array of effluents. These effluents often end up being discharged into a system of ponds that hold significant quantities of dissolved solids, salts, heavy metals, and various dissolved chemicals. Among these are hydrocarbons and treatment chemicals that can lead to substantial environmental contamination. The presence of these substances is not just a pollution risk for ponds, lakes, rivers, seas, and oceans; they can also react with the unpolluted fluids and the matrix of the surrounding disposal horizon, resulting in several undesirable environmental impacts. To effectively manage these risks, it is essential to establish a degree of compatibility between the wastewater influents and the effluents of the receiving environments. Such compatibility is crucial for significantly reducing, though not eliminating, the formation of undesirable precipitates and the inhibition of microbial activity. This is related to several interrelated factors, including (i) the chemical reactions that may occur when effluents interact with the surrounding unpolluted fluids; (ii) the physical processes of momentum, heat, and mass transfer between the influents and the receiving media; and (iii) environmental weathering conditions that may change over time. Simulation models—grounded in known

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mathematical and chemical principles—serve as valuable tools for predicting various impacts, including the effects of momentum transfer resulting from continuous wastewater discharge and the biodegradation processes of petroleum hydrocarbons within oxidation ponds. These predictive models are integral to understanding the potential behavior of different constituents as they interact with the receiving media. This understanding plays a pivotal role in determining how best to conceptualize the fate and transport of these substances at varying depths and lengths within the oxidation pond system. The stages of natural gas production and processing—extraction, separation, dehydration, and compression—produce wastewater characterized by intricate chemical compositions. This wastewater is laden with hydrocarbons and by-products such as glycols and amines, which pose severe environmental challenges when released without effective treatment. Dissolved oxygen and other reactive agents further complicate the chemical interactions taking place at the disposal sites, creating long-term threats to both surface and subsurface environmental health. To mitigate these risks, it is imperative to calculate and achieve a compatibility ratio between the influent (wastewater) and the effluent (the receiving environment). This compatibility is essential for minimizing, though not completely eradicating, the detrimental effects associated with precipitate formation, microbial inhibition, and disruptions to the delicate balance of natural ecosystems. Such risks often stem from the reactions between the effluents and unpolluted fluids, leading to alterations in chemical compositions and often resulting in precipitate formation. Physical processes—involving momentum, mass, and heat transfer—also contribute to ecological disruption by affecting how the effluents interact with the receiving environment. Weathering factors, including temperature fluctuations, wind patterns, and precipitation, can further influence this dynamic, complicating the interaction between effluent constituents and the natural environment. For instance, during periods of high precipitation, increased runoff and dilution can drastically alter the concentration and transport dynamics of pollutants, exacerbating the environmental challenges at hand. Addressing these complexities effectively requires the deployment of advanced simulation models. These models not only predict the impacts of momentum transfer due to the continuous discharge of wastewater but also analyze how this affects the biodegradation of petroleum hydrocarbons in oxidation pond systems. By closely examining these interactions, critical insights can be garnered regarding the behavior, transport, transformation, and ultimate fate of effluent constituents within the receiving medium.

Through the application of these modeling techniques, it's possible to optimize both the design and the operational strategies of wastewater treatment systems employed in natural gas production and processing facilities. Ensuring that these systems align with sustainable practices is vital for minimizing environmental impacts. Moreover, the use of these predictive models is particularly significant for conceptualizing treatment approaches that vary with changes in constituent behavior across different depths and lengths of oxidation pond systems, as well as under differing weather conditions. Ultimately, the management of effluents resulting from natural gas production and processing is not merely a regulatory or environmental obligation; it is a critical endeavor for protecting fragile ecological systems and ensuring the long-term sustainability of natural gas operations. The implementation of rigorous effluent management strategies will play a central role in promoting environmental integrity and safeguarding public health for generations to come. A model serves as a systematic device designed to represent an approximation of a particular field situation or phenomenon. In the realm of scientific and mathematical inquiry, a mathematical model plays a pivotal role by simulating field variables indirectly. It does this through the employment of governing equations that are meticulously formulated to encapsulate the processes occurring within the system being studied. Models can be categorized into three main types: predictive models, interpretative models, and generic models. Predictive models are specifically tailored to forecast the outcomes of proposed actions or interventions, allowing researchers to anticipate future scenarios based on current data. In contrast, interpretative models offer a structured framework for studying the dynamic interactions within a system, enabling a deeper understanding of the underlying mechanics. Generic models, on the other hand, focus on analyzing flow in hypothetical hydrodynamic or hydrogeologic systems, providing insights into theoretical frameworks (Ukpaka, 2011a; Efroymson, 2000; Amadi *et al*., 2007). The complexity of addressing momentum, heat, and mass transfer phenomena in liquid or porous media is particularly pronounced when conceptualizing bioremediation applications across various environments, including ponds, lakes, rivers, seas, oceans, and terrestrial landscapes. It is important to recognize that in practical applications, achieving an exact mathematical representation of a real-world situation is unattainable (Ina, 2007; Dyer *et al*., 2003; Dibble & Bartha, 1979). This is largely due to the intricate nature of fluid dynamics and the inherent properties of real liquid masses, which cannot be precisely quantified in mathematical terms. Given these complexities, there arises a pressing need to formulate a set of assumptions that facilitates a simplified understanding of the real system under consideration. Consequently, only those features and characteristics deemed relevant to the specific problems at hand are incorporated within the model's framework. A comprehensive mathematical model, along with its corresponding numerical code, is essential for simulating activities within hydrodynamic mediums. This process typically involves utilizing methodologies that yield valid computational representations for predicting how various field variables interact with hydrodynamic behaviors. Within the context of modeling, two primary components constitute the model: processes and hydrodynamic behaviors. The processes encompass both physical and chemical phenomena that occur within the system, while hydrodynamic behavior relates to the characteristics of momentum, heat, and mass within the medium, particularly in the case of petroleum hydrocarbons and microorganisms. Additional aspects, such as dissolution, transport, buoyancy flow, and precipitation, are also considered essential elements. While processes can often be constructed and rigorously studied in laboratory environments and described through mathematical equations, models of hydrodynamic behavior are inherently site and scale-dependent and must be derived from the specific conditions present in the field (Mravik *et al*., 2003; Okonji *et al*., 2008). For a research study to be deemed successful, it is crucial to ensure both accurate process identification and proper identification of the model structure. Instances where modeling results do not align with field data can often be attributed to errors in either of these two areas. It is important to note that the approach to process modeling and model structure varies significantly. Ideally, process models can be validated across different conditions, but the behaviors associated with hydrodynamic processes are usually specific to each site. Validation of the conceptual model, along with the computer code derived from it, is achieved when there is a clear confirmation that both the model and the code accurately represent the processes occurring in the real system. This validation process typically involves comparing the model's calculations with experimental measurements or observed field data, thus ensuring reliability and accuracy. Before embarking on any modeling endeavor, several critical considerations must be taken into account: (i) the observable quantities of interest that the model aims to address, (ii) the accuracy required for the predictions of these quantities, and (iii) the range of conditions under which the model has been validated. These factors are instrumental in guiding the development and application of effective models that can provide valuable insights into complex environmental systems.

2. Materials and Methods

2.1. The Organic Content Wastewater Model

In wastewater management, an essential parameter intricately linked to the levels of Dissolved Oxygen (DO) is the Biochemical Oxygen Demand (BOD). BOD is a critical metric used to assess wastewater quality, as it quantifies the amount of oxygen that microorganisms will consume while decomposing organic matter present in the water. This measurement serves as a proxy for the potential impact of the discharged wastewater on the oxygen levels of the receiving water body, which can have significant ecological implications. The relationship between wastewater discharge and the receiving waters is governed by dilution: as the volume of receiving water increases, the adverse effects of treated or untreated wastewater on the aquatic ecosystem are mitigated. The strength of wastewater is typically characterized by its effluent BOD concentration, which indicates the potential for depleting oxygen in the recipient aquatic environment. If the BOD level in the receiving water system exceeds acceptable thresholds, it signifies that the BOD strength of the effluent is excessively high, potentially harming aquatic life. The most critical parameters for evaluating wastewater quality include: Dissolved Oxygen (DO): Which indicates the amount of oxygen available for aquatic organisms. Suspended Solids (SS): Solid particles suspended in water that can affect light penetration and habitat quality. Biochemical Oxygen Demand (BOD): Measures the organic matter present in the water and its potential to affect oxygen levels. Temperature: Affects the rate of biochemical reactions and the solubility of oxygen in water. pH: Influences both microbial activity and the solubility of metals and nutrients. Nutrients (Nitrogen and Phosphorus): Essential for aquatic plants but can lead to eutrophication in excessive quantities. Heavy Metals (e.g., Mercury, Lead, Cadmium): Toxic substances that can accumulate in aquatic ecosystems and pose health risks. Hydrocarbon Composition: The presence of various hydrocarbons can impact water quality and aquatic health.

2.2. The Model for BOD Concentration

To accurately assess the BOD concentration within an aquatic system, we consider the interaction between the pollutant flow from wastewater and the natural dilution in a receiving body, such as a pond. Let (Qw) denote the flow rate of wastewater being discharged, and (beta w) represent its BOD concentration. The pond has its inflow rate, denoted as (Qp), along with its inherent BOD concentration (beta p). To predict the initial BOD concentration (beta o) of the mixed water—resulting from the interaction of the discharged wastewater and the pond—it is essential to apply the principle of mass balance. This is rooted in the conservation of mass, which states that the total mass entering a system must equal the mass accumulating within the system plus any mass leaving it. By employing this concept, the initial BOD concentration of the mixture can be determined based on the respective inflow rates and concentrations of both the wastewater and the pond water (Evans 1963). This mathematical modeling is crucial for understanding how different discharge strategies can influence the water quality of receiving bodies and for making informed decisions regarding wastewater treatment and environmental protection.

$$\beta_o = \frac{Q_p \beta_p + Q_w \beta_w}{Q_p + Q_w} \quad \text{……………. (1)}$$

Where, Q_p is the flow in the pond, Q_w is flow of wastewater discharge (m^3/s), β_p is BOD of pond upstream of discharge (mg/l) and β_w is BOD of wastewater discharge (mg/l).

2.3. Dissolved Oxygen Model

To understand the dynamics of dissolved oxygen (DO) levels, we analyze the timing and the corresponding downstream distances at which the minimum DO concentration or the highest dissolved oxygen deficit (DOC) occurs. This information is derived from various rating equations that illustrate the temporal changes in DO levels concerning wastewater discharge. This model allows us to predict when and where critical changes in oxygen levels will take place within the aquatic ecosystem.

$$Q_p\beta_p + Q_w\beta_w = (Q_p + Q_w)\beta_f \qquad \text{.............(2)}$$

$$t_c = \frac{1}{K_2 - K_1} In\left[\frac{K_2}{K_1}\left(\frac{DO_O(K_2 - K_1)}{K_1\beta_O}\right)\right] \qquad \text{................(3)}$$

$$DO_C = \frac{K_1}{K_2}\beta_o e^{-K_1 t} \qquad \text{............... (4)}$$

2.4. Relationship between Dissolved Oxygen and Biochemical Oxygen Demand

In a theoretical framework for Biochemical Oxygen Demand (BOD) over time, the oxygen consumed by microorganisms as they work to stabilize the organic matter present in wastewater sample is expressed by a specific equation (5).

$$BOD_t = \beta_o\left(1 - \ell^{-K_1 t}\right) \qquad \text{..................(5)}$$

In addition, when examining the timing and spatial distance related to the occurrence of minimum dissolved oxygen (DO) levels or peak dissolved oxygen deficits (DOC), further equations (6) and (7) come into play.

$$t_c = \frac{1}{K_2 - K_1} In\left[\frac{K_2}{K_1}\left(\frac{DO_o(K_2 - K_1)}{K_1\beta_o}\right)\right] \qquad \text{...............(6)}$$

By rearranging equation (5) to solve for βo and substituting this value into equations (6) and (3.15), we can derive new equations (8) and (9). The formulation of equation (8) is particularly valuable as it establishes a correlation between dissolved oxygen levels and biochemical oxygen demand.

$$DO_C = \frac{K_1}{K_2}\beta_O\ell^{-K_1 t} \qquad \text{...........................(7)}$$

2.5. Re-aeration Model for Surface Layers of Wastewater Influenced by Atmospheric Oxygen

In the context of deoxygenation, the rate at which reaeration occurs is modeled as a first-order reaction, which relies on the current DO deficit, denoted as D. This relationship is articulated in equation (10),

$$R_R = K_2 D \qquad \text{................(10)}$$

Where D is defined as the difference between DO saturation (DO5) and the actual dissolved oxygen (DO), measured in mg/l. The standard DO saturation value at 20°C is 9.2 mg/l, while K2 represents the reaeration constant expressed in day^-1. It's important to note that the value of K2 can fluctuate significantly, typically ranging over an order of magnitude. For instance, in a stagnant pond, K2 values may fall between 0.1 to 0.23, as noted by Metcalf & Eddy (1991), who proposed a general formula for K2 based on two parameters: the mean longitudinal velocity (in m/s) and the water depth (in meters), as described in equation (11).

$$K_2 = \frac{3.9\overline{U}^{1/2}}{H^{3/2}} \qquad \text{.............(11)}$$

Where $\overline{U}$ the mean longitudinal velocity (m/s) and H is is water depth (m)

2.6. Sedimentation Model: Impact of Solid Settling on BOD Reduction

When suspended solids are discharged into a wastewater pond, they may settle to the pond's bottom over time. This process naturally reduces the overall contaminant load being introduced to the receiving water. The rate of sedimentation is directly proportional to both the concentration of the suspended solids and their settling velocity.

$$R_s = \frac{v}{H} C_{SS} \qquad \text{................(12)}$$

This relationship can be expressed with equation (12), where the settling rate (in mg/L per day) is influenced by the settling velocity (in m/day) and the concentration of suspended solids (in mg/L). Given the typically low settling velocities, laboratory tests are often employed to determine these values, which generally fall between 0.1 and 10 m/day, as referenced by Metcalf and Eddy (1991). It is worth mentioning that the contribution of sedimentation to the mass balance equation from equation (9) is not commonly factored in.

2.7. BOD Oxidation and Surface Reaeration Model

This study expands the understanding of momentum transfer caused by the continuous discharge of wastewater, alongside the biodegradation of petroleum hydrocarbons, to incorporate additional significant parameters. The model is premised on the notion that both deoxygenation and reaeration occur as first-order processes with constant rates, encapsulated in equation (13).

$$\frac{dDO}{dt} = \frac{k_1\beta}{dexoygenation} - \frac{k_2\,DO}{reaeration} \qquad \text{..........(13)}$$

During this investigation, it was observed that a portion of the BOD is removed via sedimentation. Thus, the model can be expressed as indicated in equation (14), where Ks signifies the rate constant for BOD reduction due to sedimentation (in day^-1). Employing Laplace transformation to mathematically resolve equation (14) leads us to an expression for dissolved oxygen over time, shown in equation (16).

$$\frac{d\beta}{dt} = (k_1 + k'_s)\beta \qquad \text{...............(14)}$$

Where K_s is the rate constant for BOD removed by sedimentation (day $^{-1}$), thus, resolving equation (14) mathematically using Laplace transformation yields

$$\beta_t = \beta_o \ell^{-(k_1+k'_s)t} \qquad \text{................(15)}$$

$$DO_{(t)} = \frac{k_1\beta_o}{k_2 - (k_1 + k_s)}\left[e^{-(K_1+K'_S)t} - e^{-K_2 l}\right] + DO_o e^{-K_2 t} \qquad \text{.....................(16)}$$

Assuming there is no dissolved oxygen present during sedimentation and photosynthesis does not occur, equations (13) and (14) can be adjusted to yield a modified expression for BOD (equation 17) and related dissolved oxygen parameters (equation 18). Here, K1β represents deoxygenation, K2DO accounts for reaeration, and P stands for photosynthesis.

$$\frac{d\beta}{dt} = -\left(k_1 + k'_s\right)\beta + B \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Where B is BOD from sediment (from the benthic deposits – mg/l day)

$$\frac{dDO}{dt} = K_1\beta - K_2DO - P \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Where $K_1\beta$ is deoxygenation, K_2DO is reaeration, and P is photosynthesis, resolving equations (17) and (18) mathematically yields

$$\beta_{(t)} = \left(\beta_o - \frac{B}{K_1 + K_2}\right)e^{-(K_1+K'_s)t} + \frac{B}{K_1 + K'_s} \qquad \ldots\ldots\ldots\ldots\ldots(19)$$

$$\text{DO (t)} = \frac{K_1}{K_2 - \left(K_1 - K_s\right)}\left(\beta_o - \frac{B}{K_1 + K_S}\right)\left[e^{-(K_1+K'_s)t} - e^{-K_2 l}\right]$$

$$+ \frac{K_1}{K_2}\left(\frac{B}{K_1 + K_S} - \frac{P}{K_1}\right)\left(1 - e^{-K_2 t}\right) + DO_o e^{-k_2 t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Further mathematical resolution of equations (17) and (18) gives rise to equations (19) and (20). In scenarios where degradation is ineffective, particularly within the system's non-effective zone, BOD persists within the wastewater as a result of benthic deposits without any reduction from settling. Consequently, the parameter Ks equals zero (Ks=0), reducing equations (19) and (20) to simpler forms identified in equations (21) and (22).

$$\beta_{(t)} = \left(\beta_o - \frac{B}{K_1}\right)e^{-K_1 t} + \frac{B}{K_1} \qquad \ldots\ldots\ldots\ldots\ldots(21)$$

$$\text{DO (t)} = \frac{K_1}{K_2 - K_1}\left(\beta_O - \frac{B}{K_1}\right)e^{-K_1 t} - e^{-K_2 t} + \frac{B - P}{K_2}\left(1 - e^{-K_2 t}\right) + DO_o e^{-K_2 t} \qquad \ldots\ldots\ldots\ldots(22)$$

2.8. Mode for Correlation of BOD Initial Concentration for Continuous Discharge of Effluent into the Oxidation Pond System.

The mathematical model for determination of BOD initial concentration for continuous discharge of influences in the oxidation pond system is given as;

$$\text{BOD}_{(t)} = \left[\frac{Q_p\beta_p + Q_w\beta_w}{Q_p + Q_w}\right]\left[1 - e^{-K_1 t}\right] \qquad \ldots\ldots\ldots\ldots\ldots(23)$$

2.9. Model for Correlation of Initial BOD Concentration in Continuous Discharge into the Oxidation Pond System

The mathematical model for determining the initial concentration of BOD resulting from continuous discharge into an oxidation pond system is described by equation (23).

Recall equation (16) and making DO $_{(t)}$ the subject of the formula yields;

$$\text{DO}_{(t)} = \frac{K_1\beta_o}{K_2-K_1}\left(e^{-K_1t}-e^{-K_2t}\right)+DO_e e^{-K_2t} \qquad \text{…………(24)}$$

Therefore, substituting equation (23) into equation (24) yields

$$\text{DO (t)} = \frac{K_1}{K_2-K_1}\left(\frac{Q_p\beta_p+Q_w\beta_w}{Q_p+Q_w}\right)\left(e^{-K_1t}-e^{-K_2t}\right)+DOe^{-K_2t} \qquad \text{……………..(25)}$$

2.10. Model Correlation of Initial BOD Concentration, Dissolved Oxygen, and Biochemical Oxygen Demand

Referring back to equation (16) and manipulating it to isolate DO (t), we have the expression shown in equation (24). By substituting equation (23) into equation (24), we derive a comprehensive relationship for DO (t) as outlined in equation (25).

Recalling equation (4), which its mathematical expression is given as

$$DO_C = \frac{K_1}{K_2}\beta_o e^{-K_1t} \qquad \text{……………(26)}$$

Therefore, substituting equation (1) into equation (26) yields

$$\text{DO}_C = \frac{K_1}{K_2}\left(\frac{Q_p\beta_p+Q_w\beta_w}{Q_p+Q_w}\right)\left(e^{-K_1t}\right) \qquad \text{…………(27)}$$

2.11. Model for Correlation of Dissolved Oxygen with Distance and Initial BOD Concentration

Revisiting equation (4), which is mathematically represented in equation (26), we can substitute equation (1) into (26) to establish a correlation for DOC, articulated in equation (27).

$$\text{DO (t)} = \frac{K_1}{K_2-\left(K_1-K_S\right)}\left(\frac{Q_p\beta_p+Q_w\beta_w}{Q_p+Q_w}-\frac{B}{K_1+K_S}\right)\left(e^{-(K_1+K_S)t}-e^{-K_2t}\right)$$

$$+\frac{K_1}{K_2}\left(\frac{B}{K_1+K_S}-\frac{P}{K_1}\right)\left(1-e^{-K_2t}\right)+DO_o e^{-K_2t} \qquad \text{…………..(28)}$$

The equation (28) is useful for only the effective zone of biodegradation (progressive phase). And for the non-effective zone of biodegradation, the equation (29) becomes:

$$\text{DO (t)} = \frac{K_1}{K_2-K_1}\left(\frac{Q_p\beta_p+Q_w\beta_w}{Q_p+Q_w}-\frac{B}{K_1+K_S}\right)\left(e^{-K_1t}-e^{-K_2t}\right)+\frac{B-P}{K_2}\left(1-e^{-K_2t}\right)+DO_o e^{-K_2t} \qquad \text{……..(29)}$$

2.12. Model for Correlation of Initial BOD Concentration with BOD Oxidation and Surface

The mathematical representation derived for BOD oxidation and surface reaeration is encapsulated in equation (20). By substituting equation (1) into this equation, we arrive at a new expression for DO (t), captured in equation (28), which is applicable solely within the effective zone of biodegradation (progressive phase). For the ineffective zone, equation (29) is adjusted to reflect the conditions of that area.

2.12.1. Momentum Conservation Model for a Fluid Element in a Pond

The conservation of momentum can be expressed either in linear or angular forms. Momentum itself is defined as the product of mass and velocity. However, here we focus specifically on deriving this concept to determine the dynamics at play in the pond system. Ukpaka, *et al.*, (2010). In examining the forces acting on a fluid element consisting of petroleum hydrocarbons and other components over a specified length, denoted as Δ, we can mathematically express this analysis through a series of equations.

$$\text{Gravity } F_g = \rho g A \Delta x S_o \quad \text{…………(30)}$$

$$\text{Friction } F_f = \rho g A \Delta x S_f \quad \text{…………..(31)}$$

$$\text{Hydrostatic } F_1 - F_2 = \tfrac{1}{2}\rho g \frac{\partial}{\partial x}\left(\bar{y}^2 A\right) \quad \text{……..(32)}$$

The gravitational force, represented by (Fg), as expressed in equation (30). Similarly, the frictional force, denoted as (Ff), is detailed in equation (31). The hydrostatic pressure difference between the two points in the fluid, denoted as (F1 - F2), is given in equation (32). Here, (Delta) signifies the length of the fluid element in meters (m), while (A) refers to the average cross-sectional area measured in square meters (m^2). The slopes of the energy line and the water surface respectively, represent the rate of change in elevation concerning the horizontal distance, measured in meters per meter (m/m). The centroid height of the hydrostatic force is also noted.

$$\text{Total head} = \text{datum} + \text{water depth} + \frac{V^2}{2g} \quad \text{… (33)}$$

Where, $\frac{V^2}{2g}$ = the “velocity” head.

The energy line is indicative of the total head at each section of the fluid flow, which can be broken down into its components as outlined in equation (33). This total head is a summation of the datum, the water depth, and a term representing the velocity head, which captures the kinetic energy associated with fluid motion. For this research, it is important to establish that the slopes of the energy line, the water surface, and the slopes below the water surface are not assumed to be uniform. Hypothetically, the energy line is assumed to have a slope of (Sf), while the slope beneath the water surface is marked as (S_0). The conservation of momentum within this fluid system is encapsulated in equation (34), where the forces acting on the fluid element are related through the subsequent equation (35):

$$\left(\begin{array}{l}\textit{The rate of change}\\ \textit{of momentum for}\\ \textit{the volume}\\ \textit{element}\end{array}\right) = \left(\begin{array}{l}\textit{The resultant of the}\\ \textit{forces acting on the}\\ \textit{volume element}\end{array}\right) \quad \text{…………….(34)}$$

$$\frac{dU}{dt} = F_g + (F_1 - F_2) - F_f \quad \text{…………….(35)}$$

Substituting equations (31, 32), and (33) into equation (34) yields

$$\frac{d}{dt}(U) = \rho g A \Delta x S_o + \frac{1}{2}\rho \frac{\partial}{\partial x}\left(\bar{y}^2 A\right) - \rho g A \Delta x S_f \quad \text{……………(36)}$$

Since $$\frac{d(U)}{dt} = \rho A\Delta x\left(\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x}\right) \quad \text{………………(37)}$$

Therefore, substituting equation (37) into equation (36) and rearranging the equation obtained yields

$$\rho A\Delta X\left(\frac{\partial u}{\partial t} + V\frac{du}{\partial x}\right) = \rho g A\Delta X\left(S_o - S_f\right) + ½\rho g\frac{\partial}{\partial X}\left(\overline{y^2}A\right) \quad \text{…………… (38)}$$

Since the investigation will be conducted under the following conditions, a constant cross-section and one-dimensional flow, thus equation (38) becomes

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = g\left(S_o - S_f\right) \quad \text{…………….. (39)}$$

By substituting the specified equations for friction, hydrostatic pressure difference, and total head into equation (34), we arrive at equation (36).

At X_{11} =0, t =0, U =C_n and at X11 =L, t =T ……………..(40)

Furthermore, recognizing that (F) can also be related to the previously identified variables allows us to arrive at equation (37). By substituting this back into equation (36) and carefully restructuring it, we generate equation (38), which abstracts the fluid dynamics being analyzed. For this investigation, we operate under specific conditions: a constant cross-section and a one-dimensional flow scenario. This leads to a simplified form represented in equation (39).

To apply this mathematically, we consider boundary conditions outlined in equation (40): at position (X11 = 0), time (t = 0), and velocity (U = Cn); and at position (X11 = L), time (t = T).

Through the utilization of mathematical tools, equation (39) can be further articulated as:

U =TX ………………….. (41)

Using identical boundary conditions and employing the same mathematical techniques, such as variable separation, allows us to derive further solutions from equation (39), contributing to our understanding of fluid dynamics in this context. Equation (39) is expressed by considering the following boundary condition, such as

2.13. Correlation Model of Momentum Transfer, Biochemical Oxygen Demand and Dissolved Oxygen

Considering the following assumption that the C_n = BOD = DO concentration therefore equation (52) can be written as

$$\frac{V_{ii}^{2}}{1 - V_{ii}} = g(S_o - S_f)L_{ii}\frac{In\,BOD}{U_{Lii}} \quad \text{……………(42)}$$

$$\frac{V^{2}}{1 - V_{n}} = g(S_o - S_f)L_{u}\frac{In\,DO}{U_{Lii}} \quad \text{………………(43)}$$

Substituting equation (1) into equation (53) we have

$$\frac{V_{ii}^{2}}{1 - V_{ii}} = g(S_o - S_f)L_{u}\; In\frac{\left(Q_P\,\beta_P + \ell w\right)/Q_p + (Q_w)}{U_{Ln}} \quad \text{…………..(44)}$$

$$\frac{V_{ii}^{\;2}}{1-V_{ii}} = g(S_o - S_f)L_u \frac{In\,\beta_o}{U_{Ln}} \qquad \text{..................(45)}$$

Substituting equation (9) into equation (54) we have

$$\frac{V_{ii}^{\;2}}{1-V_{ii}} = g(S_o - S_f)L_{ii} \frac{In\,\frac{K_1}{k_2}\,\beta_o e^{-kt}}{U_{Lii}} \qquad \text{............(46)}$$

$$\frac{V_n^{\;2}}{1-V_{ii}} = g(S_o - S_f)L_n \frac{In\,\frac{k_1}{k_2}\,(Q_P\,\beta_p + \beta_w / Q_p + Q_w)e^{-kt}}{U_{Lii}} \qquad \text{...............(47)}$$

2.14. Application of Biochemical Oxygen Demand and Dissolved Oxygen Model

In this research work, we have developed comprehensive kinetic models that effectively predict both the specific growth rates of microorganisms and the specific rates of substrate utilization during the biodegradation of petroleum hydrocarbon mixtures within an oxidation pond system. These models consider the dynamics of momentum transfer resulting from the continuous discharge of wastewater, which significantly influences the biodegradation process. The applications of these models are extensive and include the following areas: Monitoring and Predicting Microbial Growth Rates: The models allow for real-time assessment and forecasting of microbial growth trajectories, thereby ensuring effective management of biodegradation processes. Analyzing Biochemical Oxygen Demand (BOD) and Dissolved Oxygen (DO) Rates: The ability to monitor and predict the specific rates of BOD and DO provides critical insights into the aquatic ecosystem's health and the efficiency of the biodegradation process. (Nwosi & Wosu 2025b) Evaluating Momentum Transfer Effects: The models facilitate the analysis of how variations in momentum transfer impact the overall biodegradation efficiency, which is essential for optimizing treatment systems. Enhancing Bioremediation Programs: Utilizing the predictive capacities of the models helps in designing and improving bioremediation strategies, ensuring that they are both effective and economical. Performance Assessment of Processes and Products: The models serve as a benchmark for evaluating the qualification of bioremediation processes and the effectiveness of various treatment products. BOD and DO Concentration Distribution:

2.15. Application of Momentum Transfer on Biodegradation of Petroleum Hydrocarbon

The generated models are integral to understanding the role of momentum transfer in the biodegradation of petroleum hydrocarbons in wastewater treatment systems. Their applications include: Performance Monitoring of Wastewater Degraders: The models help in tracking the efficiency and effectiveness of biological agents responsible for wastewater treatment. Biomass Quantification in Wastewater Ponds: They provide essential metrics for quantifying and characterizing the types and amounts of biomass that develop within treatment ponds. Kinetic Study of Bioreactor Systems: (Nwosi & Wosu, 2025a) The insights gained from these models facilitate the kinetic analysis of various bioreactor types, including different pond systems. Microbial Growth Rate Prediction: They enable accurate monitoring and forecasting of the specific growth rates of microorganisms involved in the biodegradation process. Planning Bioremediation Programs: The models can guide the design and implementation of effective bioremediation strategies tailored to specific environmental conditions. Distribution Rate Monitoring of BOD and DO Concentrations: The ability to predict how BOD and DO concentrations are distributed within a bioreactor supports operational efficiency. Substrate Rate Predictions: Forecasting the specific rates of substrate utilization enriches our understanding of how efficiently microorganisms are degrading pollutants. Assessing BOD and DO Concentration Impact: Finally, these models are instrumental in predicting how fluctuations in BOD and DO concentrations affect the biodegradation processes of individual hydrocarbons in an oxidation pond system.

2.16. Experimental Analysis

2.16.1. BOD and DO Analysis

Table 1 Standard Methods for Measuring BOD and DO in Wastewater Treatment

Parameter	Analytical Method
Biochemical Oxygen Demand (BOD) (mg/L)	APHA 5210D
Dissolved Oxygen (DO) (mg/L)	APHA 5230D

2.16.2. Reanalysis & Recommendations

Clarify Measurement Units: Ensure "mg/L" is consistently used for both parameters. Standard References: Confirm the accuracy of APHA methods (e.g., APHA Standard Methods for the Examination of Water and Wastewater). Expanded Detail *(if needed)*: Consider including testing conditions, such as incubation time (for BOD) or temperature (for DO), to enhance technical depth. In this section, we explore the methodologies for conducting biochemical oxygen demand (BOD) and dissolved oxygen (DO) analyses. This analysis is vital for determining the organic matter content in wastewater and assessing the aerobic biological treatment processes. The BOD test measures the amount of oxygen consumed by microorganisms during the decomposition of organic matter, while the DO measurement evaluates the oxygen available in the water for aquatic life and microbial processes. Together, these parameters provide crucial data for monitoring and enhancing the biodegradation of petroleum hydrocarbons in wastewater treatment systems.

2.17. Sampling

Samples were systematically collected at designated points within the oxidation pond using an automatic pipette, which was calibrated with a slide rule to accurately measure the different depths of the water. This sampling took place in one of the oxidation ponds located in the Niger Delta region of Nigeria. The collection strategy involved taking samples from various depths and locations throughout the pond system to ensure a comprehensive analysis of the wastewater. Once collected, the wastewater samples were carefully transferred into properly labeled sample bottles. To preserve the integrity of the samples during transport, they were stored in a cool container, preventing any changes to their composition prior to laboratory analysis. In addition to the depth-specific samples, surface samples were also obtained at varying distances across the pond to capture any potential variations in water quality due to influences like sunlight exposure and evaporation. The analysis aimed to assess key parameters, including the concentration of hydrocarbons (denoted as o), the presence of particulate matter (represented as), and the measurement of wave formations. These wave formations are indicative of disturbances caused by the continuous discharge of wastewater into the pond, which significantly affects the flow dynamics within the system. The direction of flow was also noted, as it plays a crucial role in understanding the dispersion of contaminants throughout the pond environment.

3. Results and Discussion

The results obtained from the investigation are presented sequentially as follows;

Wet Season Theoretical Concentration of BOD, DO o of Sub-Surface Versus Velocity

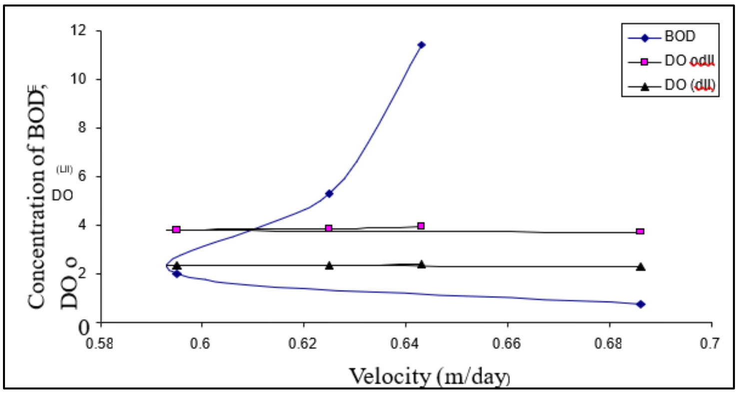

Figure 1 Wet Season Theoretical Concentration of BOD, DO o of Sub-surface Versus Velocity

Figure 1 illustrates a comparison between the theoretical concentrations of DO at two different points—DOLII and DOtII—at the surface against various sampling distances over time. The results indicate a close alignment, suggesting that the effects of momentum transfer can be interpreted through either time or distance, effectively capturing the underlying processes involved.

Furthermore, when we compare Figure 1 with Figure 4, which focuses on the wet season's theoretical concentrations of DOLII and DOtII in terms of both distance and time, we observe a consistent pattern: an increase in distance or time correlates with a decrease in dissolved oxygen levels. Interestingly, the experimental results obtained during the wet season appear to support a more favorable biodegradation process than those from the dry season. In Figure 3, we delve into the wet season's theoretical concentrations of DOLII and DOtII at various velocities. The findings reveal that velocity significantly impacts the system, acting as a function of both time and distance. It is evident that as velocity increases, the concentration of dissolved oxygen diminishes over time and distance. Notably, at a velocity of 0.443 m/day, both DOLII and DOtII reach an intersecting point of 0.18 mg/L; similarly, at 0.502 m/day, the levels drop to 0.03 mg/L, and further decline to 0.01 mg/L at a velocity of 0.5 m/day.

3.1. Wet Season Theoretical Concentration of BOD, DOo

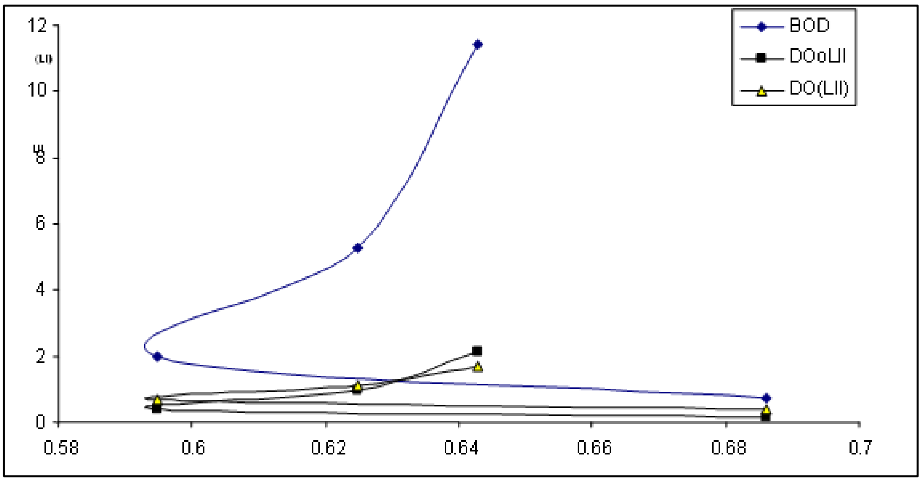

Figure 2 Wet Season Theoretical Concentration of BOD, Doo

3.2. Wet Season Theoretical Concentration of BOD and DO (tII) of Sub-Surface

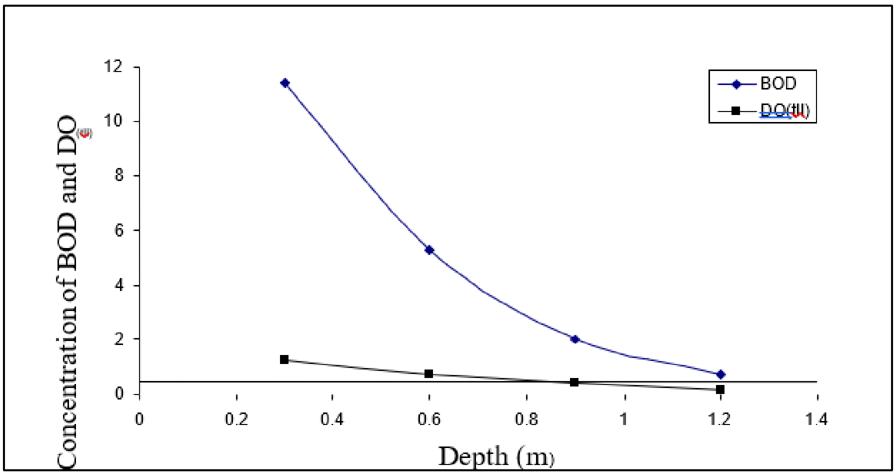

Figure 3 Wet Season Theoretical Concentration of BOD and DO (tII) of Sub-Surface

3.3. Wet Season Theoretical Concentration of BOD and DO

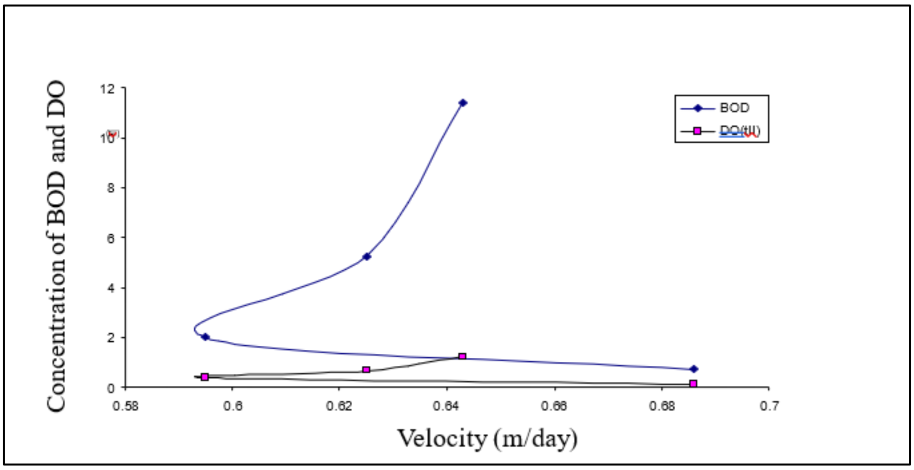

Figure 4 Wet Season Theoretical Concentration of BOD and DO

3.4. Wet Season Momentum Transfer Values of Sub-Surface Displacement Versus Distance for U

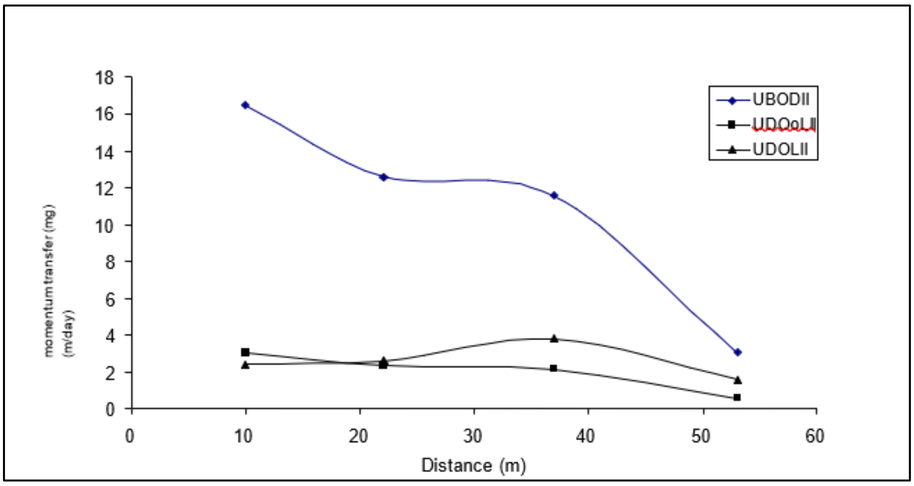

Figure 5: Wet Season Momentum Transfer Values of Sub-Surface Displacement Versus Distance for U

3.5. Wet Season Momentum Transfer -Surface Displacement

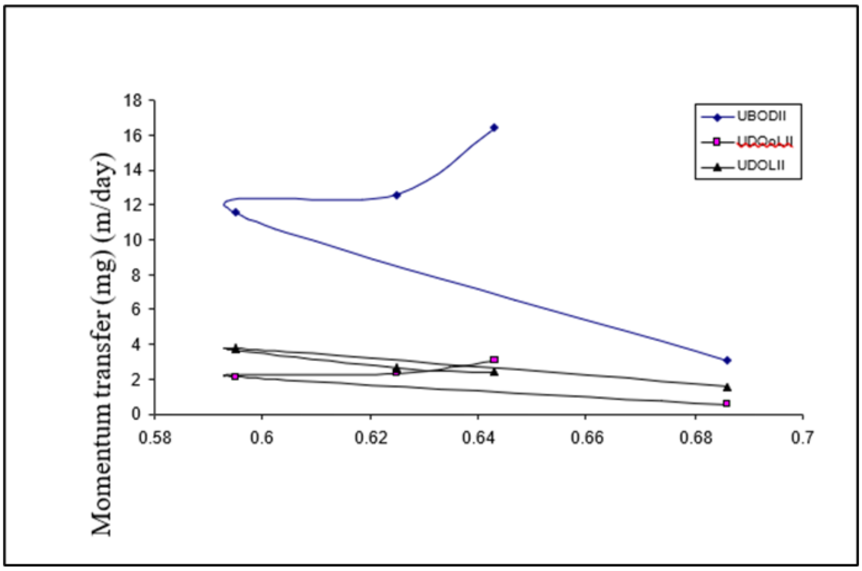

Figure 6 Wet Season Momentum Transfer -Surface Displacement

3.6. Wet Season Momentum Transfer Values of Sub-Surface Displacement

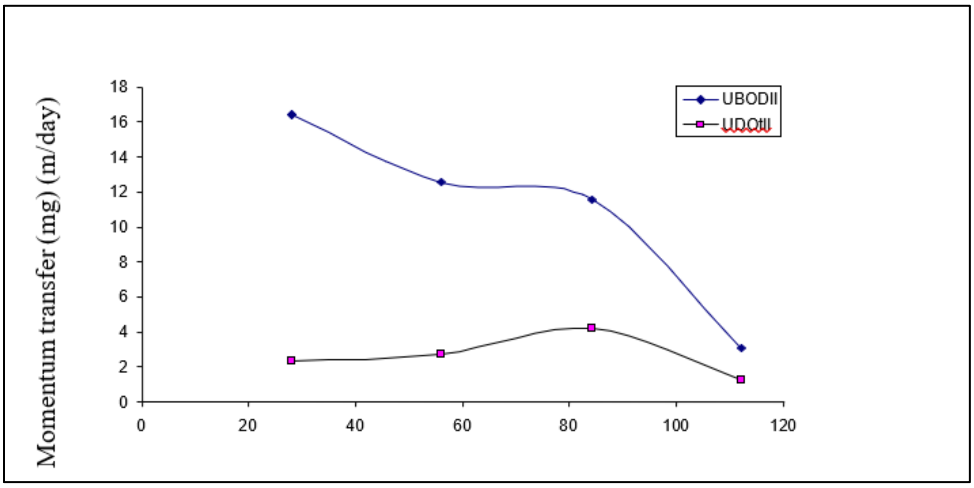

Figure 7 Wet Season Momentum Transfer Values of Sub-Surface Displacement

3.7. Wet Season Momentum Transfer Values of Surface Displacement

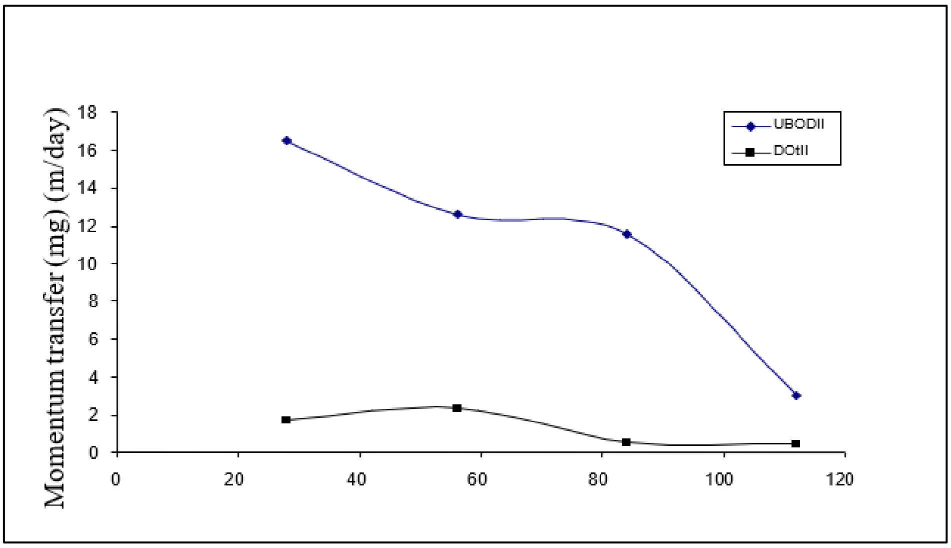

Figure 8 Wet Season Momentum Transfer Values of Surface Displacement

Wet Season Determination of Theoretical Concentration of Biochemical Oxygen Demand (BOD) and Dissolved Oxygen at Different Vertical Depths, Surface Distances, and Time. In this research, we formulated theoretical kinetic models to analyze several physicochemical parameters involved in the biodegradation of petroleum hydrocarbons and their byproducts within an oxidation pond system. The focus was on understanding the influence of momentum transfer on the concentrations of biochemical oxygen demand (BOD) and dissolved oxygen (DO). These models provide a framework for evaluating changes in dissolved oxygen levels as they relate to variations in velocity, distance, and time, as depicted in Figures 1 to 3.

Wet Season Comparison of Theoretical and Experimental Concentration of Biochemical Oxygen Demand (BOD) and Dissolved Oxygen at Different Vertical Depths, Surface Distances, Time, and Subsurface Distances. Mathematical data derived from our formulated models were systematically compared against experimental results obtained from analytical studies, yielding a commendable match. This compatibility underscores the appropriateness of these models for monitoring and predicting momentum transfer effects on the biodegradation of petroleum hydrocarbons within the oxidation pond system. Figure 4 presents the wet season theoretical concentrations of BOD and two iterations of DO (DOOdII at different levels) concerning vertical depth. It is observed that as the BOD decreases, there is also a slight decrease in dissolved oxygen, alongside an increase in vertical depth. Specifically, at a vertical depth of 0.84 m, the concentrations align at BOD = DOOdII = 2.4 mg/L. Similarly, Figure 5 highlights the general trend of decreasing concentrations of both BOD and DO with increasing vertical depth for the subsurface displacements within the pond. Figure 7 further corroborates this trend, showing a decrease in BOD concentration as subsurface depth increases, while DO concentrations exhibit minor fluctuations under changing conditions. Notably, from this figure, BOD = DOOdII = 4.0 mg/L at a specific subsurface displacement distance.

Results from Figure 6 illustrate a continued decrease in both BOD and DO concentrations with increasing distance in subsurface displacements. Meanwhile, Figure 8 indicates that BOD concentrations decline with increasing time, accompanied by slight changes in DO concentrations over the same time frame. According to Figure 7, at nine weeks into the study, both BOD and DOOdII are recorded at 4.0 mg/L, while at 11.5 weeks, they reduce to 2.4 mg/L. The findings in Figure 8 demonstrate a decreasing trend in subsurface DO concentrations as time progresses, influenced by the momentum transfer phenomenon occurring within the oxidation pond system. The results depicted in Figure 8 reveal significant decreases in both BOD and DO concentrations as velocities increase, alongside heightened depth and subsurface displacement. Notably, it is observed that BOD and DO concentrations reach equilibrium at certain velocities: for instance, both BOD and DOOdII are equal at 4.0 mg/L with a velocity of 0.603 m/day, and at 2.4 mg/L with a velocity of 0.587 m/day concerning depth displacement. In terms of time, the data shows that BOD = DOOLII = DOLII = 1.3 mg/L with a velocity of 0.63 m/day and DOOLII = DOLII = 0.7 mg/L at a velocity of 0.6 m/day for subsurface displacements. Figure 3 reiterates the observation of decreasing BOD and DO concentrations with increasing depth, as well as the impact of velocity over time. Wet Season Effect of Momentum Transfer on Biochemical Oxygen Demand (BOD) and Dissolved Oxygen (DO) Concentration. Throughout the experimental work, the influence of depth was a critical factor under investigation. As reflected in Figure 5, the effects of momentum transfer and the resulting changes in BOD and DO concentrations were meticulously analyzed, bringing insights into the dynamic interactions at play within the ecosystem of the oxidation pond.

4. Conclusion

This study investigated the development of models to examine the effects of momentum transfer and temperature on the concentrations of Biochemical Oxygen Demand (BOD) and Dissolved Oxygen (DO) in a pond system impacted by the continuous discharge of wastewater. The findings indicate significant changes in BOD and DO concentrations, highlighting the importance of understanding the interaction between effluent composition and momentum transfer dynamics. Our research emphasizes the critical need to characterize the effluent in the pond, as its composition directly influences BOD and DO levels. The data collected suggest that variations in momentum transfer and temperature can either enhance or inhibit the treatment efficiency of wastewater in the pond system. Specifically, these factors impact the biodegradation processes of various pollutants, including petroleum hydrocarbons and their metabolites, which do not contribute to inhibitory byproducts such as carbon dioxide (CO_2) or methane (CH_4). Moreover, the rate of momentum transfer observed through our experiments aligns closely with our predictive models, confirming that the proposed rate-limiting mechanism is plausible. We assumed that momentum transfer acts as a rate-controlling factor, leading to the formulation of a detailed analytical model expressed as a partial differential equation, which we subsequently solved using the method of separation of variables.

A summary of the BOD and DO properties of both influent and effluent wastewater was illustrated through graphical representations. These figures clearly demonstrated that the concentrations of BOD and DO change as functions of both displacement and time, highlighting the significance of these parameters as controlling factors in the treatment process. However, to rigorously assess whether momentum transfer and temperature are indeed the rate-limiting factors, it is essential to conduct experiments using influent wastewater that is free from biodegradation inhibitors. During field experiments, we definitively established that both momentum transfer and temperature are critical rate-limiting factors influencing wastewater treatment in ponds, particularly during the wet season. The graphical analysis further underscored that the rate of momentum transfer corresponding to BOD and DO concentrations in the pond is contingent upon mass flow, velocity, time, and spatial displacement. Our laboratory investigations revealed a consistent decrease in BOD and DO concentrations with increasing displacement and time in the wet season, attributed to effective momentum transfer within the oxidation pond system. In our pursuit to understand the intricacies of momentum transfer, we considered various factors including seasonal variations (dry and wet seasons), the presence of biodegradation inhibitors and activators, rate-limiting processes, momentum rate control, and surface reaction dynamics. The laboratory data from the wet season consistently illustrated a linear relationship in the changes of BOD and DO concentrations concerning increasing displacement and time, suggesting a second-order dependence on momentum transfer. Overall, this research offers comprehensive insights into the effect of momentum transfer on BOD and DO concentrations within pond systems, demonstrating its pivotal role in the wastewater treatment process. The implications of these findings underscore the necessity for ongoing research to optimize treatment strategies in varying environmental conditions.

Compliance with ethical standards

Acknowledgments

The authors wish to thank the management and technical staff of the department of Petroleum and Gas Engineering, Federal University Otuoke, Bayelsa State, Nigeria for granting the authors access to their laboratories.

Disclosure of conflict of interest

No conflict of interest to be disclosed

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